COMPUTER ENGINEERING

DISCRETE MATHEMATICS

SEM – 3 (CBCGS DEC- 18)

Q1] a) Two dice are rolled, find the probability that the sum is (6)

1. Equal to 1 2. Equal to 4 3. Less than 13

Solution:-

Two dice are rolled

Sample space = $6^2 = 36$

Possible combinations

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

1. Sum of roll of die = 1

   Since minimum sum when two dice are rolled = 2

   Therefore, P(sum of roll of dice = 1) = $\frac{\text{no of possible outcomes}}{\text{total no of outcomes}} = \frac{0}{36}$

   P(sum of roll of dice = 1) = 0

2. Sum of roll of die = 4

Therefore, P(sum of roll of dice = 4) = $\frac{\text{no of possible outcomes}}{\text{total no of outcomes}} = \frac{3}{36} = \frac{1}{12}$

P(sum of roll of dice = 4) = $\frac{1}{12}$

3. Sum of roll of die = 13

   Maximum sum when two dice are rolled = 12

   Therefore, P(sum of roll of dice = 13) = $\frac{\text{no of possible outcomes}}{\text{total no of outcomes}} = \frac{0}{36}$

   P(sum of roll of dice = 13) = 0

Q1] b) Use the laws of logic to show that (6)

$[(p \to q) \wedge \sim q] \to \sim p$ **is a tautology**

Solution:-

LHS:- $[(p \to q) \wedge \sim q] \to \sim p$

$\sim [(p \to q) \wedge \sim q] \vee \sim p$ .... $[p \to q = \sim p \vee q]$

$\sim [(\sim p \to q) \wedge \sim q] \vee \sim p$ .... $[p \to q = \sim p \vee q]$

$\sim [(\sim p \to \sim q) \vee (q \wedge \sim q] \vee \sim p$ .... [distributive]

$\sim [(\sim p \wedge \sim q) \vee F] \vee \sim p$ .... $[p \wedge \sim p = F]$

$\sim [(\sim p \wedge \sim q)] \vee \sim p$ .... $[p \wedge F = p]$

$[\sim(\sim p) \wedge \sim(\sim q)] \vee \sim p$ .... $[\sim(p \cap q) = \sim p \vee \sim q]$

$[p \vee q] \vee \sim p$ .... $\sim(\sim p) = p$

$(p \vee \sim p) \vee q$ ....[associative]

$T \vee q$ .... $[p \vee \sim p = T]$

T

$[(p \to q) \wedge \sim q] \to \sim p$ **is a tautology**

Q1] c) Determine the matrix of the partial order of divisibility on the set A. Draw the Hasse diagram of the poset. Indicate those which are chains **(8)**

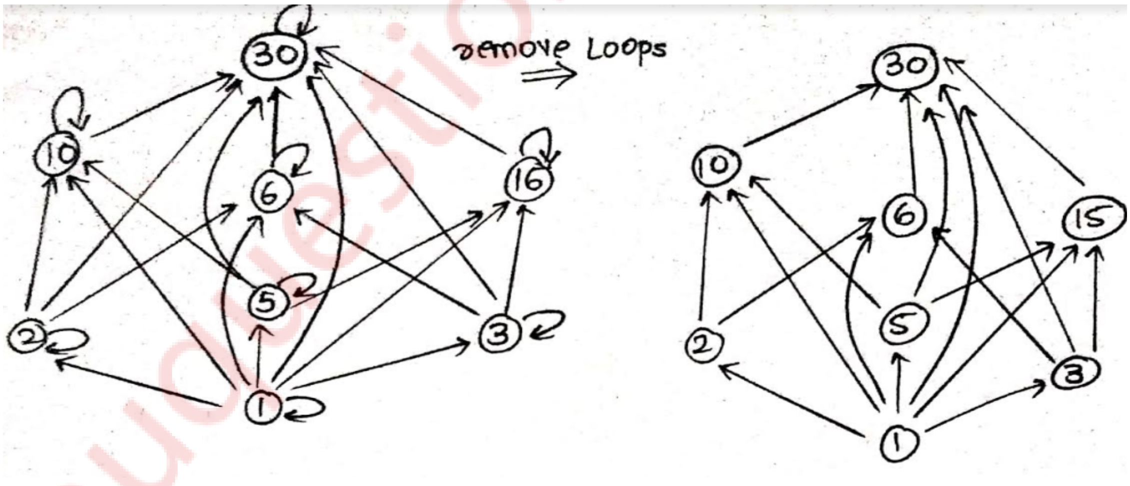
1. A = {1,2,3,5,6,10,15,30}
2. A = {3,6,12,36,72}

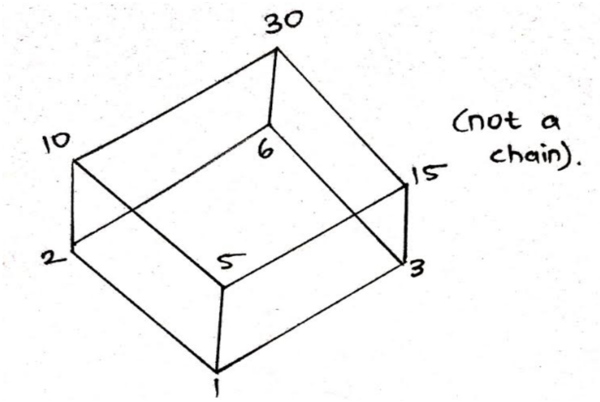
Solution:-

1. A = {1,2,3,5,6,10,15,30}

$$M_A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

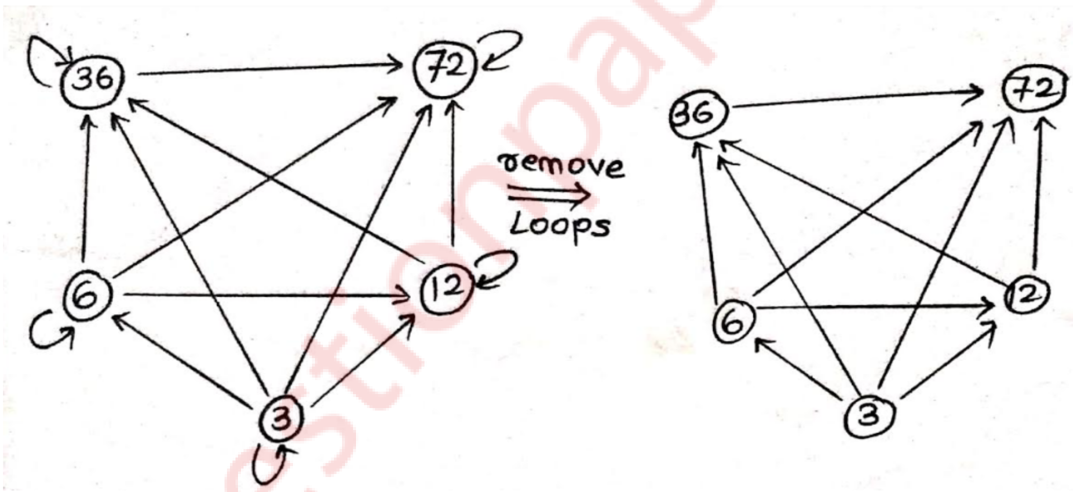
Hasse diagram:-

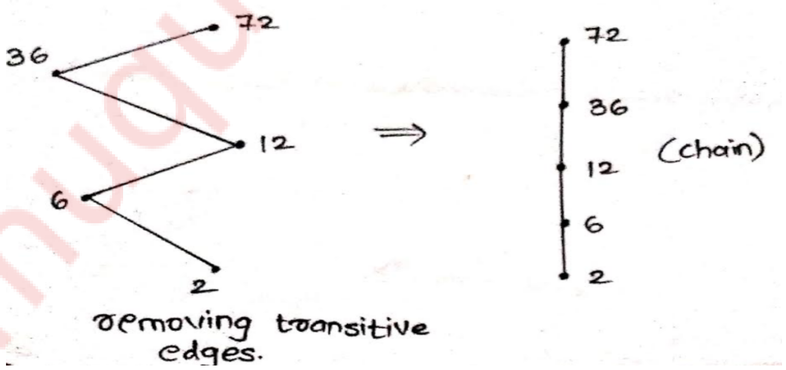




2. A = {3,6,12,36,72}

$$M_A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$





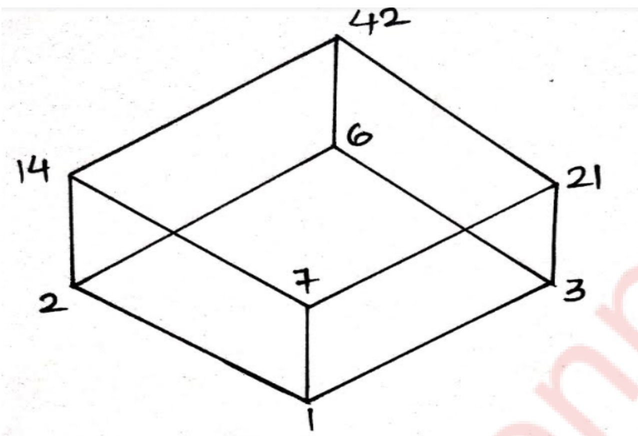
Q2] a) Find the complement of each element in D_{42} **(6)**

Solution:-

D_{42} = {1,2,3,6,7,14,21,42}

$$M = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Hasse diagram:-



Element	Complement
1	42
2	21
3	14
6	7
7	6
14	3
21	2
42	1

The given lattice D_{42} is complemented lattice.

Q2] b) Let Q be the set of positive rational numbers which can be expressed in the form 2^a3^b where a and b are integers. Prove that algebraic structure (Q, .) is a group. Where . is a multiplication operation. (6)

Solution:-

Since for every pair a,b ∈ Z ; there exists a unique element

Associative :- $(a \,.\, b).\, c = (a).\,(b \,.\, c)$

LHS:-

$[2^a3^b \,.\, 2^c3^d] \,.\, (2^e3^f)$

$[2^{a+c}3^{b+d}] \,.\, (2^e3^f)$

$2^{a+c+e}3^{b+d+f}$ .... (1)

RHS:-

$(2^a3^b).\,(2^c3^d \,.\, 2^e3^f)$

$(2^a3^b).\,(2^{c+e}3^{d+f})$

$2^{a+c+e}3^{b+d+f}$ .... (2)

Equation (1) is equal to equation (2)

‘ . ‘ is associative

Identity element :- a . e = a

$2^a \,.\, 3^b \,.\, (e) = 2^a3^b$

e = 1

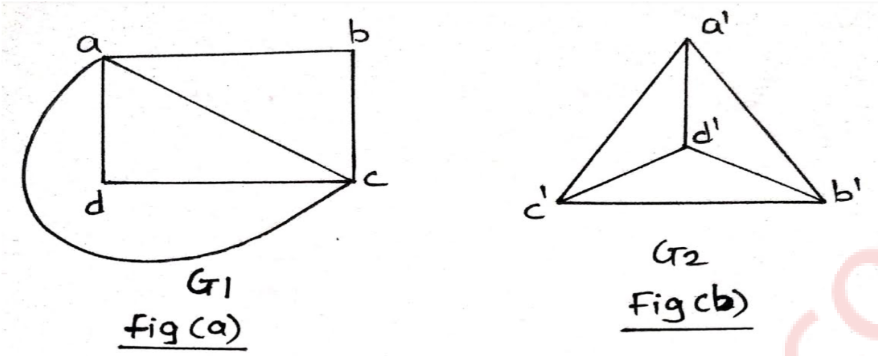
Inverse :- $a \,.\, a^{-1} = e$

$2^a3^b \,.\, a^{-1} = 1$

$a^{-1} = 2^{-a}3^{-b} \in Q$

‘ . ‘ is a group

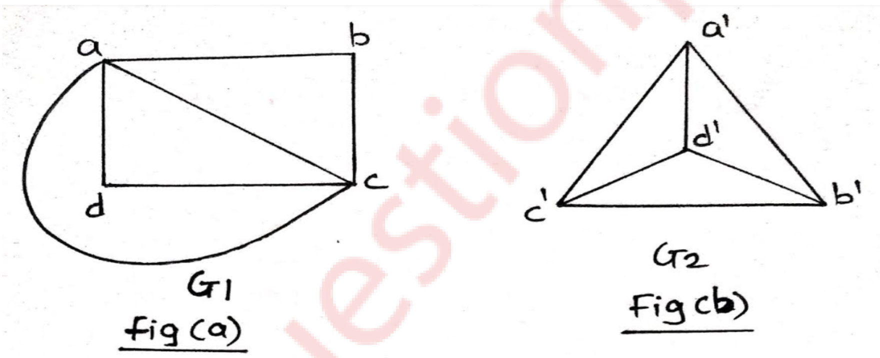
Q2] c)Define isomorphic graphs. Show whether the following graphs are isomorphic or not. (8)



Solution:-

Two graphs are said to be isomorphic if:

1. They have same number of vertices
2. They have same number of edges
3. They must have the same degree of vertices.



$n(G_1) = 4$

$n(G_2) = 4$

no of edges in $G_1 = 6$

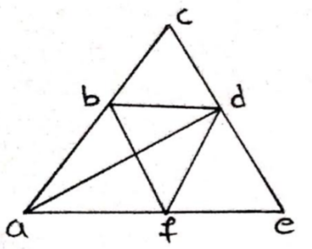
no of edges in $G_2 = 6$

degree of a = 4 ,degree of a' = 3

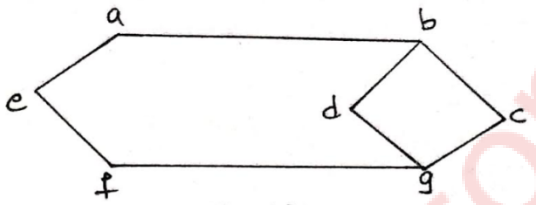
degree of G_1 and G_2 are not equal

hence, G_1 and G_2 are not isomorphic.

Q3] a) Determine which of the following graph contains an Eulerian or Hamiltonian circuit. **(6)**



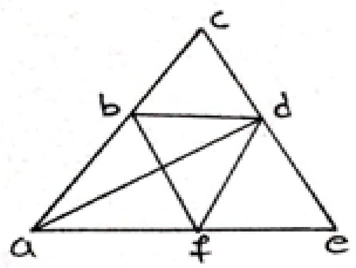
Fig(a)



Fig(b).

Solution:-

Fig (a)



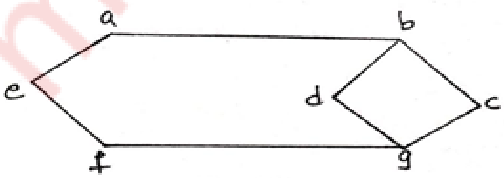
Fig(a)

Not Eulerian as degree of a is 3 which is odd

Yes Hamiltonian circuit is present

a - b - c - d - e - f

Fig (b)



Fig(b).

Not Eulerian as b and g are of odd degree not Hamiltonian as each vertex cannot be visited.

Q3] b) For all sets A,X and Y show that (6)

$$A \times (X \cap Y) = (A \times X) \cap (A \times Y)$$

Solution:-

TP = $A \times (X \cap Y) = (A \times X) \cap (A \times Y)$

LHS:-

$= A \times (X \cap Y)$

$= \{(x,y) \mid x \in A \text{ and } y \in X \cap Y\}$

$= \{(x,y) \mid x \in A \text{ and } (y \in X \text{ and } y \in Y)\}$

$= \{(x,y) \mid (x \in A \text{ and } y \in X) \ (y \in X \text{ and } y \in Y\}$

$= \{(x,y) \mid (x \in A \text{ and } y \in X\} \cap \{(x,y) \mid x \in A \text{ and } y \in Y\}$

$= (A \times X) \cap (A \times Y)$

= RHS

Q3] c) Let f(x) = x + 2, g(x) = x – 2 and h(x) = 3x for x∈ R, where R = set of real numbers. Find (gof), (fog), (fof), (gog), (foh),(hog), (hof), (fohog) (8)

Solution:-

1. gof = g(f(x)) = g(x+2) = (x+2) – 2 = x
2. fog = f(g(x)) = f(x-2) = (x-2) + 2 = x
3. fof = f(f(x)) = f(x+2) = x+2+2 = x+4
4. gog = g(g(x)) = g(x-2) = (x-2)-2 = x-4
5. foh = f(h(x)) = f(3x) = (3x) + 2 = 3x+2
6. hog = h(f(x)) = h(x+2) = 3(x+2) = 3x+6
7. fohog = f[h(g(x))] = f[h(x-2)] = f[3(x-2)] = f(3x-6) = 3x-6+2 = 3x-4

Q4] a) Let R is a binary relation. Let S = {(a,b) | (a,c) ∈ R and (c,b) ∈ R for some c} show that if R is an equivalence relation then S is also an equivalence relation. (6)

Solution:-

R is equivalence relation; therefore R is reflexive, symmetric and transitive.

Let a, b, c be any three elements

By data if a R b and a R c => b R c

Putting c =a; we get ;

a R b and a R a => b R a

but by reflexive; a R a is true

if a R b; then b R a

therefore S is symmetric

if a R b and a R c then b R c

since R is symmetric if a R b, then b R a

b R a and a R c give b R c

therefore S is transitive

S is reflexive, symmetric and transitive

Therefore S is equivalence relation.

Q4] b) Determine the generating function of the numeric function a_r where

1. $a_r = 3^r + 4^{r+1}$, $r \geq 0$
2. $a_r = 5$, $r \geq 0$ **(6)**

Solution:-

1. $a_0 = 3^0 + 4^1 = 4$

   $a_1 = 3^1 + 4^2 = 19$

   $a_2 = 3^2 + 4^3 = 73$

   Generating function

   $\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \_\_\_\_\_\_\_\_\_\_$

   $= 4 + 19x + 73x^2 + \_\_\_\_\_\_$

2. For taking 5

   $a_0 = 5; a_1 = 5; a_2 = 5$; ....

   Generating function;

   $\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \_\_\_\_\_\_\_\_\_\_$

   $= 5 + 5x + 5x^2 + \_\_\_\_\_\_$

   $= 5(1 + x + x^2 .....)$

   $= 5(1 - x)^{-1}$

Q4] c) Consider the (3,6) encoding function e : $B^3 \rightarrow B^6$ defined by

e(000) = 000000 e(001) = 001100 e(100) = 100101 e(111) = 111010

e(010) = 010011 e(011) = 011111 e(110) = 110110 e(101) = 101001

decode the following words relative to a maximum likelihood decoding function

1. 000101 2. 010101 (8)

Solution:-

e : $B^3 \rightarrow B^6$

e(000) = 000000 e(001) = 001100

e(010) = 010011 e(011) = 011111

e(100) = 100101 e(101) = 101001

e(110) = 110110 e(111) = 111010

000000	001100	010011	011111	100101	101001	110110	111010
000001	001101	010010	011110	100100	101000	110111	111011
000010	001110	010001	011101	100111	101011	110100	111000
000100	001000	010111	011011	100001	101101	110010	111110
001000	000100	011011	010111	101101	100001	111110	110010
010000	011100	000011	001111	110101	111001	100110	101010
100000	101100	110011	111111	000101	001001	010110	011010
100011	101111	110000	111100	000110	001010	010101	011001

1. 000101 is received in 5th column and is underlined the word at top is 100101

   By data e(100) = 100101

   d(100101) = 100

2. 010101 is received in 7th column and is underlined. The word at top is 110110

   e(110) = 110110

   d(110110) = 110

Q5] a) Determine the number of positive integers n where $1 \le n \le 100$ and n is not divisible by 2,3 or 5. **(6)**

Solution:-

n(S) = no of integers in the set = 100

n(T) = no of integers divisible by 2 = 100/2 = 50

n(T’) = no of integers divisible by 3 = 100/3 = 33

n(F) = no of integers divisible by 5 = 100/5 = 20

n(T ∩T’) = $\frac{100}{2\times3}$ = 16;   n(T’ ∩ F) = $\frac{100}{5\times3}$ = 6

n(T ∩ F) = $\frac{100}{2\times5}$ = 10;   n(T ∩ T’ ∩ F) = $\frac{100}{2\times3\times5}$ = 3

no of integers divisible by 2 or 3 or 5

n(T ∪ T’ ∩ F) = n(T) + n(T’) + n(F) – n(T ∩ T’) – n(T ∩ F) – n(T’ ∩ F) + n(T ∩ T’ ∩ F)

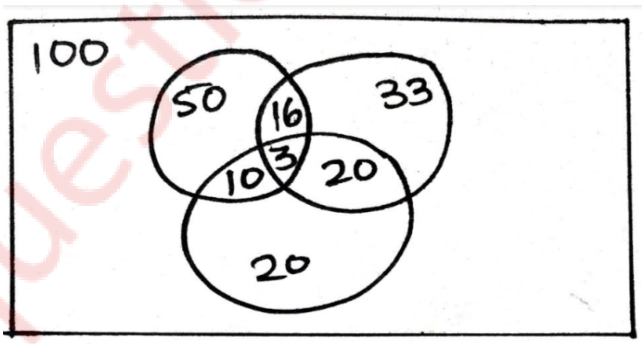
= 50 + 33+ 20 – 16 – 10 – 6 + 3

= 74

No of integers no divisible by 2 or 3 or 5

⇨ N(T∪T'∪F) = n(S) – n(T ∪ T’ ∪ F)

= 100 – 74 = 26



Q5] b) Use mathematical induction to show that

1 + 5+ 9 + …………….. + (4n-3) = n(2n-1) **(6)**

Solution:-

Let us prove P(1) is true

LHS : 4(1) – 3 = 1

RHS : 1(2- 1) = 1

LHS = RHS

P(1) is true

Let us assume P(k) is true

1 + 5 + 9 + …………. (4k- 3) = k(2k – 1)

Let us prove P(k+1) is true

1 + 5+ 9 + ……….. (4k – 3) + 4(k + 1) – 3 = (k + 1)[2(k+1)-1]

LHS : k(2k -1) + (4k + 1)

= $2k^2 - k + 4k + 1$

= $2k^2 + 3k + 1$

= $2k^2 + 2k + k + 1$

= 2k(k+1) + 1(k+1)

= (2k+1)(k+1)

= (k+1)

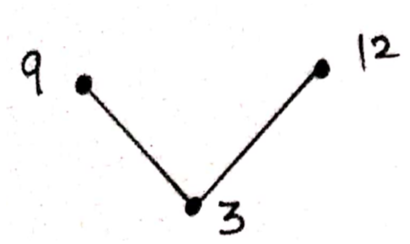
= RHS

Therefore P(k) is true for all k ∈ N

Q5] c) Find the greatest lower bound and least upper bound of the set {3,9,12} and {1,2,4,5,10} if they exists in the poset (z+, /). Where / is the relation of divisibility. (8)

Solution:-

1. S = {3, 9, 12}



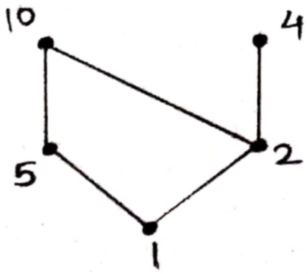
GLB:-

∧	3	9	12
3	3	3	3
9	3	9	3
12	3	3	12

LUB:-

v	3	9	12
3	3	9	12
9	9	9	-
12	12	-	12

2. S = {1,2,4,5,10}



GLB:-

∧	1	2	4	5	10
1	1	1	1	1	1
2	2	2	2	1	2
4	2	2	4	1	2
5	1	1	1	5	5
10	2	2	2	5	10

LUB:-

v	1	2	4	5	10
1	1	2	4	5	10
2	2	2	4	10	10
4	4	4	4	-	-
5	5	10	-	5	10
10	10	10	-	10	10

Q6] a) Let A = {1,2,3,4} and Let R = {(1,1), (1,2), (1,4), (2,4), (3,1), (3,2), (4,2), (4,3), (4,4)} . find transitive closure by Warshall's algorithm. (6)

Solution:-

A = {1,2,3,4}

R = {(1,1), (1,2), (1,4), (2,4), (3,1), (3,2), (4,2), (4,3), (4,4)}

$$M_R = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

$$W_0 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

$C_1 \rightarrow 1$ is at 1,3

$R_1 \rightarrow 1$ is at 1,2,4

Replace by 1 at (1,1), (1,2), (1,4), (3,1), (3,2), (3,4)

$$W_1 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

$C_2 \rightarrow 1$ is at 1,3

$R_2 \rightarrow 1$ is at 4

Replace at (1,4) and (3,4)

$$W_2 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

$C_3 \rightarrow 1$ is at 4

$R_3 \rightarrow 1$ is at 1,2,4

Replace by 1 at (4,1), (4,2) (4,4)

$$W_3 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$C_4 \rightarrow 1$ is at 1,2,3,4

$R_4 \rightarrow 1$ is at 1,2,3,4

Replace by 1 in all combinations

$$W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

R^{00} = {(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1),(3,2), (3,3), (3,4),(4,1),(4,2), (4,3), (4,4)}

Q6] b) Let H = {$[0]_6$, $[3]_6$} find the left and right cosets in group Z_6. Is H a normal subgroup of group of Z_6 (6)

Solution:-

Table of Z_6

$+_6$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

The group Z_6 is abelian as a + b = b + a for a,b ∈ Z_6

Left coset of H = {[0],[3]} with respect to a in the set Z_6 is

aH = {a * h|h ∈H}

0H = {0 + 0,0+3} = {0,3} ; 1H = {1+0,1+3} = {1,4}

2H = {2+0,2+3} = {2,5}; 3H = {3+0,3+3} = {3,0}

4H = {4+0, 4+3} = {4,1}; 5H = {5+0,5+3} = {5,2}

Right coset of H = {[0],[3]} wrt a in set

Ha = {h * a | h ∈ H}

$H0 = \{0+0,3+0\} = \{0,3\}$; $H1 = \{0+1,3+1\} = \{1,4\}$

$H2 = \{0+2,3+2\} = \{2,5\}$; $H3 = \{0+3,3+3\} = \{3,0\}$

$H4 = \{0+4,3+4\} = \{4,7\}$; $H5 = \{0+5,3+5\} = \{5,8\}$

Clearly we have,

H0 = 0H, 1H = H1, 2H = H2, ………… 5H = H5

H is a normal subgroup of Z_6

Q6] c) Find the complete solution of the recurrence relation

$a_n + 2a_{n-1} = n + 3$ for $n \geq 1$ and with $a_0 = 3$ **(8)**

Solution:-

The characteristics equation :- $r + 2 = 0$

$$r = -2$$

Solution is $a_n^{(h)} = B(-2)^n$

Let particular solution be $a_n^{(p)} = an + b$

Putting this value of a_n in given equation

$(an+b) + 2[a(n-1)+b] = n + 3$

$3an + (3b-2a) = n + 3$

$3a = 1 \Rightarrow a = \frac{1}{3}$

And $3b - 2a = 3$

$b = \frac{11}{9}$

Total solution $a_n = B(-2)^n + \frac{n}{3} + \frac{11}{9}$

Constant is found by initial condition

$3 = B + \frac{11}{9}$

$B = \frac{16}{9}$

Required solution:- $a_n = \frac{16}{9}(-2)^n + \frac{n}{3} + \frac{11}{9}$