

(3 hrs )

Total Marks : 100

**N.B. :** (1) All questions are compulsory.

- (2) **Figures** to the **right** indicate **full** marks.
- (3) Draw **neat** diagrams wherever **necessary**.
- (5) Symbols have usual meaning unless otherwise stated.
- (5) Use of **non-programmable** calculator is allowed.

CONSTANTS :  $h = 6.626 \times 10^{-34}$  Js,  $k = 1.38 \times 10^{-23}$  JK<sup>-1</sup>

1. Attempt any two:---
  - (a) Explain the terms 10
    - i) Binomial probability functions
    - ii) Chebyshev's inequality
    - iii) Laws of large number
  - (b) Explain the following random variables 10
    - i) Probability functions for the random variables.
    - ii) Mean value, Variance and Standard deviation of a random variable  $x$ .
    - iii) Cumulative distribution functions.
  - (c) Explain the Poisson distribution and derive the required relations. 10
  
2. Attempt any two:---
  - (a) State the second order non- homogeneous linear ordinary differential equation with constant variable and solve the same. 10
  - (b) What is method of separation of variables for solving partial differential equation? Explain the same by solving heat equation. 10
  - (c) What do mean by Hyperbolic function in complex number. 10

Prove the following formulas.

  - i)  $\cos 2(z) = \cos^2(z) - \sin^2(z)$
  - ii)  $\sinh 2(z) = 2\sinh(z) \cosh(z)$
  
3. Attempt any two:---
  - (a) Derive an expression of probability of occupying a given energy for dominant configuration. 10
  - (b) Define partition function. Derive expression for translation partition function. If translation partition function for Ar is equal to  $4.88 \times 10^{29}$ . Find the volume in which it confines at temperature of 298k (Given  $m_{Ar} = 6.63 \times 10^{-26}$  kg). 10
  - (c) Derive expressions of total energy  $E$  for two-level system. Determine the total energy of an ensemble consisting of  $N$  particles that have only two energy levels separated by energy  $h\nu$ . 10

4. Attempt any two:---
- (a) Using Maxwell-Boltzmann distribution law of velocities, derive the expression for root mean square velocity, average velocity and most probable velocity. **10**
  - (b) What are Fermions? Derive Fermi-Dirac distribution law. **10**
  - (c) Obtain Wien's displacement law and Stefan-Boltzmann law using Plank's radiation formula. **10**
5. Attempt any **Four**:---
- (i) Find the probability density function of exactly 54 heads in 100 tosses of a coin using Normal or Gaussian distribution. **05**
  - (ii) Three coins are tossed; what is the probability that two are heads and one tails? That the first two are heads and the third tails? If at least two are heads, what is the probability that all are heads? **05**
  - (iii) If path of particle in the (x, y) in plane is given by  $z = z(t)$   $z = 5 e^{i\omega t}$  find displacement and velocity. **05**
  - (iv) Find the value of  $\cos z = 2$ . **05**
  - (v) What is the difference in energy between the and states for molecular oxygen constrained by a one-dimensional box having a length of 1.00 cm? **05**
  - (vi) What is the weight associated with the configuration corresponding to observing 40 heads after flipping a coin 100 times? How does this weight compare to that of the most probable outcome? **05**
  - (vii) A box of area  $1 \text{ m}^2$  is divided into 100 cells of equal areas. 300 balls are thrown in this box. Calculate a priori probability that the three balls will fall in every cell. **05**
  - (viii) Show that the Plank's law and the Rayleigh-Jeans's law become identical if the temperature becomes very high. **05**

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