

Time : 3 Hrs

Marks : 80

- Question No.1 is compulsory.
- Solve ANY THREE questions from the remaining five questions.
- The figure to the right indicates full marks.
- Assume suitable data wherever required.

**Q. 1** Solve ANY FOUR questions from following.

Marks  
20

- Explain iso-parametric, sub-parametric and super-parametric element.
- Explain shape function and enlist the properties of shape functions.
- Explain h-method and p-method of FEM.
- Explain Jacobian matrix. And describe the significance of Jacobian Matrix in co-ordinate transformation.
- Explain plane stress and plane strain conditions applied to elasticity problems.
- Explain the sources of error in FEM.

**Q. 2 a)** Solve following differential equation using galerkin method

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$$3 \frac{d^2y}{dx^2} - \frac{dy}{dx} + 8 = 0 ; 0 \leq x \leq 1$$

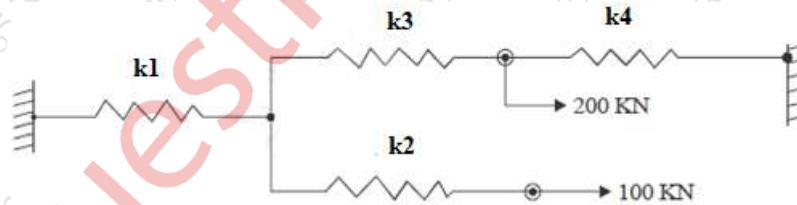
Boundary Conditions:  $y(0) = 1, y(1) = 2$ , find  $y(0.3)$

**b)** Using the concept of serendipity, derive the shape functions for four node rectangular element in natural co-ordinate system ( $\xi$  and  $\eta$ ).

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**Q. 3 a)** Determine the displacement at nodes by using principle of minimum potential energy approach

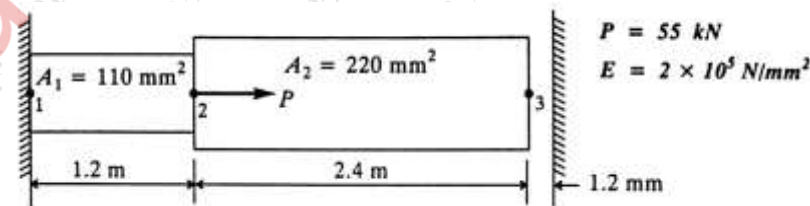
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Where,  $k_1=100 \text{ N/mm}$  ;  $k_2=300 \text{ N/mm}$  ;  $k_3=150 \text{ N/mm}$  ;  $k_4=200 \text{ N/mm}$

**b)** Determine the unknown reactions, displacement and element stresses for the stepped bar shown in the figure below.

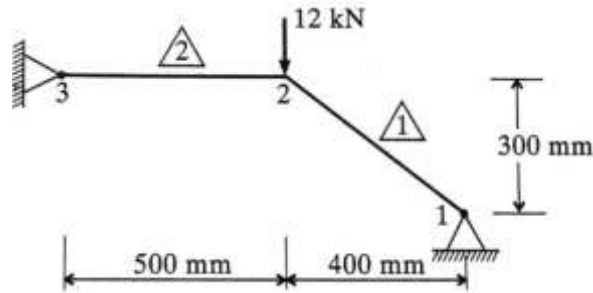
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**Q. 4 a)** Find the natural frequency of axial vibrations of a bar of uniform cross section of  $30 \text{ mm}^2$  and length of 1 meter using consistent mass matrix. Take  $E = 200 \text{ GPa}$  and density =  $8000 \text{ kg/m}^3$ . Take two linear elements.

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- b) Find nodal displacement, reaction forces and stresses in each element for a truss given below. Take  $E = 2 \times 10^5 \text{ N/mm}^2$  and  $A = 200 \text{ mm}^2$ . 10



- Q. 5 a) Calculate linear interpolation functions for linear triangular element whose vertices are A(2, 5), B(1, -1) and C(3, 4). 08
- b) Consider the steady laminar flow of a viscous fluid through a long circular cylindrical tube. The governing equation is 12

$$-\frac{1}{r} \frac{d}{dr} \left( r \mu \frac{dw}{dr} \right) = \frac{P_0 - P_L}{L} \equiv f_0$$

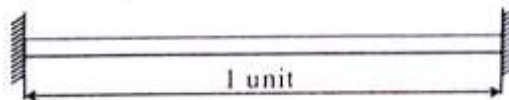
Where,  $w$  is axial ( $z$ -axis) component of velocity,  $\mu$  is the viscosity, and  $f_0$  is the gradient of pressure (includes static pressure and gravitational force). The boundary conditions are:

$$\left( r \frac{dw}{dr} \right) \Big|_{r=0} = 0, \quad w(R_0) = 0$$

Using symmetry and two linear elements or one quadratic element, determine the velocity field and compare with exact solution at nodes:

$$w_z(r) = \frac{f_0 R_0^2}{4\mu} \left[ 1 - \left( \frac{r}{R_0} \right)^2 \right]$$

- Q. 6 a) Determine the two natural frequencies of transverse vibration of a beam fixed at both ends as shown in figure. Use both Lumped and Consistent mass matrices and comment on the results. Divide the whole domain into two elements of equal lengths. [ Take  $EI = 10^6$  units,  $\rho A = 10^6$  units ] 10



- b) Derive the element matrix equation for a simple bar fixed at one end and loaded axially at the other end, as shown in figure. The cross-section area of the bar is  $A$ , and the modulus of elasticity is  $E$ . The governing differential equation is given by: 10

$$\frac{d}{dx} \left[ EA \frac{du}{dx} \right] = 0; \quad \text{for } 0 < x < l$$

