1T01434 - S.E.(Mechanical) Engineering)(SEM-IV)(Choice Base Credit Grading System) ((R-20-21) (C Scheme) / 41221 - Engineering Mathematics - IV

QP CODE: 10014587   DATE: 08/12/2022

(3 Hours)   **[Total Marks: 80]**

N.B.: 1) Question No. **1** is **Compulsory**.

2) Answer **any THREE** questions from Q.2 to Q.6.

3) Figures to the right indicate full marks.

Q.1 (a) What is the value of $\int_{0}^{1+i} \left(x - y + ix^2 \right) dz$ along the line from $z = 0 \ to \ z = 1 + i$ **(5)**

(b) Find a and b such that $\vec{F} = (axy + z^3)i + x^2 j + bz^2 xk$ is irrotational **(5)**

(c) A random variable X has probability mass function p(x) $= kx^3$; x=1,2,3,4 then find the value of k , mean, variance. **(5)**

(d) Find the probability that at most 4 defective bulbs will be found in a box of 200 bulbs if it is known that 2% of the bulbs are defective. **(5)**

Q.2 (a) Find the rank correlation coefficient between X and Y ; **(6)**

X	17	13	15	16	6	11	14	9	7	12
Y	36	46	35	24	12	18	27	22	2	8

(b) A random variable has the MGF $M_X(t) = \frac{3}{3-t}$. Find mean and Variance of X. **(6)**

(c) Obtain Laurent's series expansions of f(x) = $\frac{z-1}{z^2-2z-3}$;$|z| > 3$. **(8)**

Q.3 (a) A coin is tossed. If it turns up heads two balls are drawn from urn A otherwise two ballsare drawn from urn B. Urn A contains 3 black and 5 white balls. Urn B contains 7 blackand one white ball. What is the probability that urn A was used, given that both ballsdrawn are black? **(6)**

(b) Fit a straight line $y = a + bx$ into the given data. **(6)**

x:	10	20	30	40	50
y:	22	23	27	28	30

(c) Prove that$\overline{F} = (6xy^2 - 2z^3)i + (6x^2 y + 2yz)j + (y^2 - 6z^2 x)k$ is irrotational. Find scalar potential of $\overline{F}$. Hence find the work done of moving particle from $(1,0,2)$ to $(0,1,1)$. **(8)**

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Q.4 (a) Using Green's Theorem evaluate $\int_c (xy + y^2)dx + x^2 dy$ and c is closed curve **(6)** of the region bounded by $y = x$ and $y = x^2$.

(b) A machinist is expected to make engine parts with axle diameter of 1.75 cm. A **(6)** random sample of 10 parts shows a mean diameter of 1.85 cm, with a S.D of 0.1 cm. Based on this sample, would you say that the work of the machinist is inferior?

(c) A random variable X follows a normal distribution with mean 14 and standard **(8)** deviation 2.5 find (1)P[X<8] (2) P[X>18] (3) P[12<X<15] Given: Area between z=0 and z=2.4 is 0.4918 ; Area between z=0 and z=1.6 is 0.4452 ; Area between z=0 and z=0.8 is 0.2882 ; Area between z=0 and z=0.4 is 0.1554.

Q.5 (a) The standard deviation from two random samples of sizes 9 and 13 are 1.99 **(6)** and 1.9. Can the samples be regard as drawn from normal population with same standard deviation?$(F_{(8,12)}(0.025) = 3.51, F_{(12,8)}(0.025) = 4.20)$

(b) Use Gauss's Divergence Theorem to evaluate $\iint_s \overline{N}.\overline{F}\ ds$, where **(6)** $\overline{F} = 4xi - 2y^2 j + z^2 k$ and S is region bounded by $x^2 + y^2 = 4, z = 0, z = 4$.

(c) Obtain both Line of regressions for the data given below **(8)** Given $\sum X = 250$; $\sum Y = 300$; $\sum XY = 7900$;$\sum X^2 = 6500$; $\sum Y^2 = 10000$ and n =10 (in usual notation)

Q.6 (a) Evaluate Value of $\int_c \frac{\sin 2z\ dz}{(z + \pi/3)^4} dz$ is where C:$|z| = 2$ **(6)**

(b) The following data find the correlation coefficient to marks obtained by 11 **(6)** students in 2 tests, one held at the beginning of the year and the other at the end of the year after intensive coaching:

Test 1	19	23	16	24	17	18	20	18	21	19	20
Test 2	17	24	20	24	20	22	20	20	18	22	19

(c) A die was thrown 132 times and the following frequencies were observed. **(8)**

No. obtained	1	2	3	4	5	6	Total
Frequency	15	20	25	15	29	28	132

Test the hypothesis that the die is unbiased at 5% level of significance.

(Given: Table value of χ^2 at 5% level of significance and 5 degree of freedom is 11.07)

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