(3 Hours) **Total Marks :80**

Note: 1) Question No.1 is compulsory

2) Attempt any Three from the remaining

Q1

a) Find L[$sinh^5 t$] **5**

b) Find a , b , c, d ,e if
f(z)= (ax^3 +by^2x + 3x^2 +cy^2 +x)+i(d x^2y - 2y^3 +exy +y) is analytic **5**

c) Find half range sine series of f(x) = x(π – x) in (0 , π) **5**

d) If $A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$ Find eigenvalue of Adj(A) **5**

Q2

a) If L[f(t)] = $\frac{9s}{9s^2-3s+6}$ then find L[e^t f(3t)] **6**

b) Find Fourier series for f(x)= x^2 ; $-\pi < x < \pi$ and f(x+ 2π) = f(x) **6**

c) Find analytic function f(z)=u+iv in terms of z where
u + v = e^x (cosy + siny) **8**

Q3

a) A string is stretched and fastened to two points distance l apart. Motion is started by displacing the string in the form y=a sin(πx/ l) from which it is released at time t=0.Show that the displacement of a point at a distance x from one end at time t is given by **6**

$$y = a \sin\left(\frac{\pi x}{l}\right) \cos\left(\frac{\pi ct}{l}\right)$$

b) Prove that u=x^2 -y^2 -2xy +2x -3y is harmonic function hence find it's harmonic conjugate function. **6**

c) Find the Fourier series to represent f(x) = $\begin{cases} x\ , & 0 < x < \pi \\ 2\pi - x\ , & \pi < x < 2\pi \end{cases}$ in (0 , 2π) **8**

Q4

a) Evaluate $\int_0^\infty e^{-t}\left[\frac{\cos 6t-\cos 4t}{t}\right]dt$ **6**

b) Find inverse Laplace transform of $\frac{1}{(s-2)^2(s+1)}$ **6**

c) Is the matrix $A=\begin{bmatrix}2 & 0 & 2\\ 0 & 2 & 1\\ 0 & 0 & 1\end{bmatrix}$ diagonalizable ? If so find the Diagonal form of A and transforming matrix of A **8**

Q5

Using Cayley Hamilton Theorem find $A^9-6A^8+10A^7-3A^6+A+I$

a) where $A=\begin{bmatrix}1 & 2 & 3\\ -1 & 3 & 1\\ 1 & 0 & 2\end{bmatrix}$ **6**

b) Solve by Crank-Nicholson simplified formula $\frac{\partial^2 u}{\partial x^2}-\frac{\partial u}{\partial t}=0$, $0\le x\le 1$ subject to the condition $u(0,t)=0, u(1,t)=100$, $u(x,0)=100\,(x-x^2)$ & h=0.25 for one time step **6**

c) Find the inverse Laplace transform of **8**

(i) $\log[(s^2-4)(s^2-9)]$

(ii) $\frac{s}{(s-5)^2}$

Q6

a) Find the Laplace Transform of $\int_0^t ucoshusinhudu$ **6**

Find the solution of $\frac{\partial^2 u}{\partial x^2}-32\frac{\partial u}{\partial t}=0$, 0<x<1,

b) u(x,0)=0, u(0,t)=0, u(1,t)=10+t, taking h=0.25, k=0.025 for $0\le t\le 1$ where 'h' is the step length for x axis and 'k' is the step size in time direction using Bender-Schmidt method. **6**

c) Find inverse Laplace transform of $\frac{s}{(s^2+16)^2}$ using convolution theorem **8**
