1T01433 - S.E.(Mechanical) Engineering)(SEM-III)(Choice Base Credit Grading System) ((R- 19) (C Scheme) / 51621 - Engineering Mathematics-III

QP CODE: 10037571 DATE: 21-11-2023

Time (3 Hours) **Max. Marks: 80**

Note: (1) Question No. 1 is Compulsory.

(2) Answer any three questions from Q.2 to Q.6.

(3) Figures to the right indicate full marks.

1. (a) Find the eigen values of $A^2 + 2I$ where $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -2 & 0 \\ 3 & 5 & 3 \end{bmatrix}$ (5)

(b) Find the Laplace transform of *f(t)*, where

$$f(t) = \begin{Bmatrix} t^2, 0 < t < 1, \\ 1, \qquad t > 1 \end{Bmatrix}$$ (5)

(c) Determine the constants a, b, c, d if

$f(z) = x^2 + 2axy + by^2 + i(cx^2 + 2dxy + y^2)$ is analytic. (5)

(d) Obtain half range Sine Series for f(x) $= x^2$, in 0 < x < 3. (5)

2.(a) Find $L^{-1}\left(\frac{4s+12}{(s^2+8s+12)}\right)$ (6)

(b) Find the Laplace transform of $e^{-4t}\int_0^t u \ \sin 3u \, du$. (6)

(c) Obtain the Fourier expansion for $f(x) = \begin{Bmatrix} \pi x & 0 \le x \le 1 \\ \pi(2-x) & 1 \le x \le 2 \end{Bmatrix}$ and

hence deduce that

$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$.. (8)

3(a) Use Cayley- Hamilton theorem to find $2A^4 - 5A^3 - 7A + 6I$ where $A = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$. (6)

(b)Determine the solution of one-dimensional heat equation

$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ under the boundary conditions u (0, t) =0, u (*l*, t) =0 and u (x, 0) =x,

(0<x<*l*), *l* being the length of the rod. (6)

(c) Using Convolution theorem find the inverse Laplace transform of $\frac{(s+2)^2}{(s^2+4s+8)^2}$ (8)

4 (a) Find the orthogonal trajectory of the family of curves given by $x^3y - xy^3 = c$ (6)

(b) Prove that $\int_0^{\infty} e^{-t} \frac{sin^2 t}{t} dt = \frac{1}{4}\log 5$. (6)

(c) Using Crank-Nicholson formula, Solve $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0. u(0,t) = 0, u(4,t) = 0,$

$u(x,0) = \frac{x}{3}(16 - x^2)$. Find u_{ij} for i=0, 1, 2, 3, 4 and j=0, 1, 2 taking $h = 1$. (8)

5 (a) Find inverse Laplace transform of $\log\left(\frac{s^2+a^2}{\sqrt{s+b}}\right)$. (6)

(b) Show that the function $u = \frac{1}{2}\log(x^2 + y^2)$ is harmonic and find its corresponding analytic function and its harmonic conjugate. (6)

(c) Solve $\frac{\partial^2 u}{\partial x^2} - 32\frac{\partial u}{\partial t} = 0$ by Bender-Schmidt method, subject to the conditions, $u(0,t) = 0$, $u(x,0) = 0$, $u(1,t) = t$, taking h=0.25, 0<x<1. (8)

6 (a) Using Laplace transform Evaluate

$$\int_0^{\infty} e^{-2t}\left(\int_0^t \frac{e^{-u}\sin u}{u} du\right) dt$$ (6)

(b) Obtain Fourier series for $f(x) = x\cos x$ in $(-\pi, \pi)$. (6)

(c) Show that the matrix $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ is diagonalisable. Find the diagonal form D and the diagonalising matrix M. (8)
