(Time: 3 hours) Max. Marks: 80

N.B. (1) Question No. 1 is compulsory.

(2) Answer any three questions from Q.2 to Q.6.

(3) Use of Statistical Tables permitted.

(4) Figures to the right indicate full marks.

Q1 a) If $A = \begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix}$ then find the Eigen values of $A^3 + 6A^{-1} + 2I$ [5]

b) Evaluate $\int_0^{1+i} (x^2 + iy)\, dz$,along the path $(i) y = x$, $(ii)\ y = x^2$ [5]

c) Write the dual of the following problem [5]

$$\text{Maximise} \quad z = 3x_1 + 10x_2 + 2x_3$$
$$subject\ to \quad 2x_1 + 3x_2 + 2x_3 \leq 8$$
$$3x_1 - 2x_2 + 4x_3 = 4$$
$$x_1, x_2, x_3 \geq 0$$

d) A certain drug administered to 12 patients resulted in the following change in their Blood Pressure

5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4 [5]

Can we conclude that drug increase the Blood Pressure?

Q2 (a) Using Cauchy's residue theorem evaluate [6]

$\int_C \frac{1-2z}{z(z-1)(z-2)} dz$, Where c is |z|=1.5

(b) Verify Cayley-Hamilton theorem and find A^{-1} for $A = \begin{bmatrix} 1 & 8 \\ 2 & 1 \end{bmatrix}$. Hence, find $2A^3 - A^2 - 35A - 44I$. [6]

(c) Solve by Simplex Method [8]

$$Maximise \quad z = 4x_1 + 10x_2$$
$$\text{Subject to} \quad 2x_1 + x_2 \leq 50$$
$$2x_1 + 5x_2 \leq 100$$
$$2x_1 + 3x_2 \leq 90$$
$$x_1, x_2 \geq 0$$

Q3 a) Based on the following data determine if there is a relation between literacy and smoking

	Smokers	Non-smokers	[6]
Literates	83	57	
Illiterates	45	68	

(Given that Critical value of chi-square 1 d. f and 5% L.O.S is 3.841

b) Obtain Laurent's series expansion of $f(z) = \frac{1}{z^2+4z+3}$ [6]

$when \quad (i)\ |z| < 1 \quad (ii)\ 1 < |z| < 3 \quad$ (ii) $|z| > 3$

c) Using the method of Lagrangian multipliers solve the following N.L.P.P [8]

Optimise $z = {x_1}^2 + {x_2}^2 + x_3^2$

Subject to $x_1 + x_2 + 3x_3 = 2$

$5x_1 + 2x_2 + x_3 = 5$

$x_1, x_2, x_3 \geq 0$

Q4a) Using the method of Lagrange's multipliers solve the following N.L.P.P [6]

Optimise $z = {x_1}^2 + {x_2}^2 + {x_3}^2 - 10x_1 - 6x_2 - 4x_3$

Subject to $x_1 + x_2 + x_3 = 7$

$x_1, x_2, x_3 \geq 0$

b) Find the inverse Z-transform of $\frac{1}{z^2-3z+2}$, $if\ ROC\ is \quad (i) \quad |z| < 1 \quad (ii)\ |z| > 2$ [6]

c) Show that the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ is diagonalizable. Find the transforming matrix and the diagonal matrix. [8]

Q5a) Find $Z\{f(k) * g(k)\} \quad if \quad f(k) = \left(\frac{1}{2}\right)^k \ , g(k) = cos\pi k$ [6]

b) Find the Eigen values and Eigen Vectors of the following matrix. [6]

$$A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$$

c) Solve by the dual Simplex Method [8]

$Minimise\ z = x_1 + x_2$

Subject to $2x_1 + x_2 \geq 2$

$-x_1 - x_2 \geq 1$

$x_1, x_2 \geq 0$

Q6a) Find $Z\{2^k \cos(3k+2)\}, k \geq 0.$ [6]

b) If the heights of 500 students is normally distributed with mean 68 inches and standard deviation 4 inches, estimate the number of students having heights (i) greater than 72 inches (ii) less than 62 inches (iii) between 65 and 71 inches [6]

c) Using Kuhn Tucker conditions, solve the following NLPP [8]

$Maximise \quad z = 2{x_1}^2 - 7{x_2}^2 + 12x_1x_2$

Subject to $2x_1 + 5x_2 \leq 98$

$x_1, x_2 \geq 0$