1T01233 - S.E.(Information Technology Engineering)(SEM-III)(Choice Base Credit Grading System)
(R- 19) (C Scheme) / 51421 - Enginering Mathematics III
QP CODE : 10037856   DATE: 21-11-2023

(Time: 3 Hours)   **Max. Marks: 80**

N.B.   (1) Question No. 1 is compulsory.
       (2) Answer any three questions from Q.2 to Q.6.
       (3) Use of Statistical Tables permitted.
       (4) Figures to the right indicate full marks

Q1.

(a) Find the Laplace transform of $\frac{\cos 2t \sin t}{e^t}$ [5]

(b) Find k such that $f(z) = \frac{1}{2} \log(x^2 + y^2) + i \tan^{-1}\frac{kx}{y}$ is analytic   [5]

(c) Calculate the Spearman's rank correlation coefficient R   [5]
 X   : 10 , 12 , 18 , 18 , 15 , 40 .
 Y   :   12 , 18, 25 , 25 , 50 , 25 .

(d) Find the inverse Laplace transform of $\log\left(\frac{s^2 + a^2}{s^2 + b^2}\right)$.   [5]

Q2.
(a) A continuous random variable has probability density function
          $f(x) = k(x - x^2)$, $0 \le x \le 1$.
                          $f(x) = 0$ otherwise
      Find k , mean and variance.   [6]

(b) Find the Laplace transform of $e^{-3t} \int_0^t u \sin 3u \, du$.   [6]

(c) Obtain the Fourier series to represent f (x) = x^2 in (0, 2π)

   Hence show that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \ldots\ldots$ [8]

Q3.
(a) If the imaginary part of the analytic function w $= u + i v = f(z)$ is   [6]
    $V = x^2 - y^2 + \frac{x}{x^2 + y^2}$, then show that $u = -2xy + \frac{y}{x^2 + y^2}$.

(b) Find inverse Laplace transform of $\frac{2s^2 - 6s + 5}{(s^3 - 6s^2 + 11s - 6)}$ [6]

(c) Fit a second-degree parabolic curve and estimate y when x = 10
    x   : 1 ,   2 , 3 , 4 , 5 ,   6 ,   7 ,   8 ,   9 ,
    y   : 2 , 6 , 7 , 8 , 10 , 11 , 11 , 10 , 9 .   [8]

Q4.
(a) Obtain the Fourier series to represent f (x) = x^3 in ($-\pi, \pi$).   [6]

(b) Find (i) the equation of the lines of Regression (ii) coefficient of correlation for
     the following data
     X :   65 ,   66 ,   67 ,   67,   68 ,   69 , 70 , 72 .
     Y: 67 ,   68 ,   65 , 66 ,   72 ,   72 , 69 , 71 .   [6]

(c) Prove that $\int_0^\infty e^{-\sqrt{2}t} \frac{\sin t \sin ht}{t} \, dt = \frac{\pi}{8}$.   [8]

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Q5.

(a) Find the orthogonal trajectories of the family of curves $x^3y - xy^3 = c$. [6]

(b) Find the moment generating function of the distribution

X	:	-2	3	1
P X = x)	:	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{6}$

hence find first four central moments . [6]

(c) Obtain the half range cosine series of $f (x) = x$ in $(0, 2)$

Hence show that $\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4}$ …….. [8]

Q6.(a) Using convolution theorem Find the inverse Laplace transform of $\left[\frac{s^2}{(S^2+2^2)^2}\right]$ [6]

(b) The probability density function of a random variable X is

X	:	1	2	3	4	5	6	7
P (X =x) :		k	2k	3k	k^2	k^2 + k	2 k^2	$4k^2$

Find k , p (X < 5), P (X > 5) [6]

.(c) If $v = 3 x^2 y + 6 x y - y^3$,show that v is harmonic function

And find the corresponding analytic function . [8]

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