1T01233 - S.E.(Information Technology Engineering)(SEM-III)(Choice Base Credit Grading System) (R- 19) (C Scheme) / 51421 - Enginering Mathematics III

QP CODE: 10011988 DATE: 21/11/2022.

(Time: 3 hours) **Max. Marks: 80**

N.B. (1) Question No. 1 is compulsory.

(2) Answer any three questions from Q.2 to Q.6.

(3) Use of Statistical Tables permitted.

(4) Figures to the right indicate full marks.

Q1 (a) Find Laplace transform of $\frac{\cos\sqrt{t}}{\sqrt{t}}$ given that $L\{\sin\sqrt{t}\} = \frac{\sqrt{\pi}}{2s^{3/2}}\, e^{-(1/4s)}$ [5]

(b) Calculate Spearman's rank correlation coefficient for the following data: [5]

X	32	55	49	60	43	37	43	49	10	20
Y	40	30	70	20	30	50	72	60	45	25

(c) Find inverse Laplace transform of $\frac{2s-1}{s^2+8s+29}$ [5]

(d) If $f(z)= qx^2y+2x^2+ry^3-2y^2-i(px^3 -4xy-3xy^2)$ is analytic, find the values of p, q, and r [5]

Q2 (a) Find Laplace transform of e^{3t} f(t) where $f(t)=\begin{cases} t-1, & 1<t<2 \\ 3-t, & 2<t<3 \\ 0, & otherwise \end{cases}$ [6]

(b) Two unbiased dice are thrown. If X represents sum of the numbers on the two dice. Write probability distribution of the random variable X and find mean, standard deviation , and $P(|X-7|\geq 3)$ [6]

(c) Obtain Fourier series for $f(x) = x \sin x$ in the interval $0 \leq x \leq 2\pi$. [8]

Q3 (a) Using Milne-Thompson's method construct an analytic function $f(z)= u+ iv$ in terms of z where $u+v = e^x(\cos y+\sin y) + \frac{x-y}{x^2+y^2}$ [6]

(b) Using convolution theorem find the inverse Laplace transform of $\frac{(s+3)^2}{(s^2+6s+5)^2}$ [6]

(c) Fit a parabola $y= a+bx+cx^2$ to the following data and estimate y when x=10 [8]

x	1	2	3	4	5	6	7	8	9
y	2	6	7	8	10	11	11	10	9

Q4 (a) Find Laplace transform of $e^{-(1/2)t}\, t\, f(3t)$ if $L\{f(t)\} = \frac{1}{s\sqrt{s+1}}$ [6]

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(b) Find half range sine series for $f(x) = x - x^2$, $0 < x < 1$. [6]

Hence deduce that $\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} \ldots = \frac{\pi^3}{32}$

(c) Given regression lines 6y=5x+90, 15x=8y+130, $\sigma_x{}^2 = 16$. [8]
Find i) $\bar{x}$ and $\bar{y}$, ii) r, iii) $\sigma_y{}^2$ and iv) angle between the regression lines

Q5 (a) Can the function $u = r + \frac{a^2}{r}\cos\theta$ be considered as real or imaginary part of an analytic function? If yes, find the corresponding analytic function. [6]

(b) An unbiased coin is tossed three times. If X denotes the absolute difference between the number of heads and the number of tails, find moment generating function of X and hence obtain the first moment about origin and the second moment about mean. [6]

(c) Evaluate $\int_0^\infty e^{-2t} \cosh t \int_0^t u^2 \sinh u \cosh u \, du \, dt$ [8]

Q6 (a) Find inverse Laplace transform of $\frac{1}{(s-2)^4(s+3)}$ using method of partial fractions. [6]

(b) If a continuous random variable X has the following probability density function [6]

$$f(x) = \begin{cases} k\, e^{-\frac{x}{4}}, & \text{for } x > 0 \\ 0, & \text{elsewhere} \end{cases}$$ find k, mean and variance.

(c) Find half range cosine series for $f(x) = x$, $0 < x < 2$. [8]

Hence deduce that i) $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \frac{1}{7^4} + \ldots = \frac{\pi^4}{96}$

ii) $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \ldots = \frac{\pi^4}{90}$

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