Q.P. Code :23005

[Time: Three Hours] **[Marks:80]**

Please check whether you have got the right question paper.

N.B:
1. Question no. 1 is compulsory.
2. Attempt any three of the remaining.
3. Figures to the right indicate full marks.

Q.1 a) Find the Laplace transform of e^{-4t} sinh t sin t. 05

b) Find half-range sine series for $f(x) = \frac{\pi}{4}$ in $(0, \pi)$. 05

c) Find the values of Z for which the following function is not analytic.
Z= sin hu cos v + i cos hu sinv. 05

d) Show that $\nabla\left[\frac{(\bar{a}\cdot\bar{r})}{r^n}\right] = \frac{\bar{a}}{r^n} - \frac{n(\bar{a}\cdot\bar{r})\bar{r}}{r^{n+2}}$, where $\bar{a}$ is a constant vector. 05

Q.2 a) Find the inverse Z- transform of $F(z) = \frac{1}{(z-3)(z-2)}$ if $|z| < 2$. 06

b) Verify Laplace's equation for $u = \left(r + \frac{a^2}{r}\right)\cos\theta$ also find v and f(z). 06

c) Find the Fourier series for the periodic function 08

$$f(x) = \begin{cases} -\pi & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

State the value of f(x) at x=0 and hence, deduce that

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$$

Q.3 a) Find $L^{-1}\left[\frac{1}{(S-3)(S-3)^2}\right]$ using convolution theorem. 06

b) Show that the set of functions sin x , sin 2x , sin 3x, -------------- is orthogonal on the interval $[0,\pi]$ 06

c) Verify Green's Theorem for $\int_C \bar{F}\cdot d\bar{r}$ where $\bar{F} = x^3 i + xyj$ and c is the triangle whose vertices are (0,2) , (2,0) and (4,2) . 08

Q.4

a) Find Laplace transform of $f(t) = \begin{cases} a \sin pt, & 0 < t < \frac{\pi}{p} \\ 0, & \frac{\pi}{p} < t < \frac{2\pi}{p} \end{cases}$ 06

and $f(t) = f\left(t + \frac{2\pi}{p}\right)$.

b) Show that $\bar{F}$ = $(y^2-z^2+3yz-2x)i + (3xz+2xy)j + (3xy-2xz+2z)k$ is both solenoidal and irrotational. 06

c) Find half range cosine series for f(x) =x , 0<x<2. 08

Hence deduce that $\frac{\pi^4}{90} = \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + - - - - -$

Q.5

a) Show that $\iint_S (\nabla r^n).d\bar{s} = n(n+1) \iiint_V r^{n-2}\, dv$ using Gauss's Divergence theorem. 06

b) Find the Z-transform of $\{k^2 e^{-ak}\}$, $k \geq 0$. 06

c) (i) Find $L^{-1}\left[\frac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)}\right]$ 08

(ii) Find $L^{-1}\left[\frac{s^2+a^2}{\sqrt{s+b}}\right]$

Q.6

a) Use Laplace transform to solve, 06

$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 8y = 1$ where, y(0) =0 , y'(0) = 1

b) Find the bilinear transformation which maps the points z=∞, i, 0 onto the points 0,i,∞ respectively of w-plane. 06

c) Express the function $f(x) = \begin{cases} \frac{\pi}{2}, & for\ 0 < x < \pi \\ 0, & for\ x > \pi \end{cases}$ 08

for Fourier Sine Integral and Show that

$\int_0^\infty \frac{1-\cos\pi w}{w} \sin wx\ dw = \frac{\pi}{2}$ when $0<x<\pi$

************All THE BEST************