



Total Marks: 80
 Duration: 3Hrs

NB 1. Question No.1 is compulsory

2. Attempt any three from the remaining six questions
3. Figures to the right indicate full marks

Q1a If the Laplace Transform of $e^{-t} \int_0^t u \cos 2u \, du$ [20]

b Prove that $f(z) = \sinh z$ is analytic and find its derivative

c Obtain Half range Sine Series for $f(x) = x+1$ in $(0, \pi)$

d Find a unit vector normal to the surface $x^2 y + 2xz = 4$ at $(2, -2, 3)$

Q2 a Prove that $\vec{F} = (2xy^2 + yz)\vec{i} + (2x^2 y + xz + 2yz^2)\vec{j} - (2y^2 z + xy)\vec{k}$ is Irrotational.

Find Scalar Potential for \vec{F}

[6]

b Find the inverse Laplace Transform using Convolution theorem

$$\frac{(s-1)^2}{(s^2 - 2s + 5)^2}$$

[6]

c. Find Fourier Series of $f(x) = \begin{cases} \pi x; & 0 \leq x \leq 1 \\ \pi(2-x); & 1 \leq x \leq 2 \end{cases}$

[8]

Q3 a Find the Analytic function $f(z) = u + iv$ if $v = \frac{x}{x^2 + y^2} + \cosh x \cos y$

[6]

b Find Inverse Z transform of $\frac{(3z^2 - 18z + 26)}{(z-2)(z-3)(z-4)}$, $3 < |z| < 4$

[6]

c Solve the Differential Equation $\frac{d^2 y}{dt^2} + 2 \frac{dy}{dx} + 2y = 5 \sin t$, $y(0) = 0$, $y'(0) = 0$ using Laplace Transform

[8]

Q4 a Find the Orthogonal Trajectory of $3x^2 y - y^3 = k$

[6]

b Find the Z-transform of $2^K \sinh 3K$, $K \geq 0$

[6]

c Express the function $f(x) = \begin{cases} 1 & ; |x| < 1 \\ 0 & ; |x| > 1 \end{cases}$ as Fourier Integral. Hence evaluate $\int_0^\infty \frac{\sin \lambda}{\lambda} \cdot \cos(\lambda x) \, d\lambda$

[8]

- Q5 a Evaluate using Stoke 's theorem $\int_C (2x - y)dx - yz^2 dy - y^2 z dz$ where C is the circle $x^2 + y^2 = 1$ corresponding to the sphere $x^2 + y^2 + z^2 = 1$ above the XY plane [6]
- b Show that $w = \frac{2z + 3}{z - 4}$ maps the circle $x^2 + y^2 - 4x = 0$ into straight line $4u + 3 = 0$ [6]
- c Find Inverse Laplace Transform i) $e^{-s} \tanh^{-1} s$ ii) $\frac{6}{(2s + 1)^5}$ [8]
- Q6 a Find the Laplace transform of $f(t) = \frac{2t}{3}, 0 \leq t \leq 3, f(t+3) = f(t)$ [6]
- b Find Complex Form of Fourier Series for $\sin(\alpha x); (-\pi, \pi), \alpha$ is not an integer [6]
- c Verify Green's theorem for $\int_C (2x^2 - y^2)dx + (x^2 + y^2)dy$ where C is the boundary of the surface enclosed by lines $x=0, y=0, x=2, y=2$ [8]
