

Q.P. Code : 23178

[Time: Three Hours]

[Marks:80]

Please check whether you have got the right question paper.

- N.B:
1. Question.No.1 is compulsory.
 2. Attempt any three from the remaining six questions.
 3. Figures to the right indicate full marks.

- Q.1 *Find the*
 a) ~~If the~~ Laplace transform of $\sin^2 3t$ 20
 b) Prove that $f(z) = \log z$ is analytic
 c) Obtain Fourier series for $f(x) = x^2$ in $(-2,2)$
 d) Find the Z-Transform of $\cos 2k, k \geq 0$
- Q.2
 a) Prove that $\vec{F} = 2xyz^3i + x^2z^3j + 3x^2yz^2k$ is irrotational. 06
 Find Scalar potential for \vec{F}
 b) Find the inverse Laplace Transform using Convolution theorem 06
 $\frac{1}{(s^2+6s+18)^2}$
 c) Find Fourier Series of $f(x) = \frac{\pi-x}{2}$ in $(0,2\pi)$. 08
 Hence deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \dots$.
- Q.3
 a) Find the Analytic function $f(z) = u + iv$ if $u + v = \cos x \cosh y - \sin x \sinh y$ 06
 b) Find Inverse Z transform of $\frac{2z^2-10z+13}{(z-3)^2(z-2)}$, $2 < |z| < 3$ 06
 c) Solve the Differential Equation $\frac{d^2y}{dt^2} + 2\frac{dy}{dx}y = 3te^{-1}$, $y(0) = 4, y'(0) = 2$ using Laplace Transform 08
 $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 3te^{-1}$
- Q.4
 a) Find the Orthogonal Trajectory of $x^2 + y^2 - 3xy + 2y = c$ 06
 b) Using Greens theorem evaluate $\int_C (x^2 - y)dx + (2y^2 + x)dy$, C is closed path formed 06
 by $y = 4, y = x^2$

- c) Express the function $f(x) = \begin{cases} \sin x; & 0 < X \leq \pi \\ 0 & ; X > \pi \end{cases}$ as Fourier Integral. Hence evaluate 08

$$\int_0^{\infty} \frac{\cos(\lambda \pi / 2)}{1 - \lambda^2} d\lambda$$

- Q.5
- a) Find Inverse Laplace Transform of $\frac{2s^2 - 6s + 5}{s^3 - 6s^2 + 11s - 6}$ 06
- b) Find the Bilinear Transformation that maps the points $z = 1, i, -1$ into $w = i, 0, -i$ 06
- c) Evaluate using Stoke's theorem $\int_c \vec{F} \cdot \vec{dr}$ where c is the boundary of the circle $x^2 + y^2 + z^2 = 1, z = 0$ and $\vec{F} = yzi + zxj + xyk$ 08

- Q.6
- a) Find the Directional derivative of $\phi = x^2 + y^2 + z^2$ in the direction of the line $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$ at $(1, 2, 3)$ 06
- b) Find complex form of Fourier series for $e^{ax}; (-\pi, \pi)$ 06
- c) Find Half Range sine Series for $f(x) = x(2 - x)$ $0 < x < 2$ 08
 hence deduce that $\sum \left(\frac{1}{n^2}\right) = \frac{\pi^6}{945}$
