

SE comp

SE IT III

Applied Mathematics - III 31 May 2014

(CBGS)

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QP Code : NP-18619

(3 Hours)

[Total Marks : 80

- N.B. : (1) Question No.1 is compulsory.
(2) Attempt any three questions from Question No.2 to Question No.6.
(3) Non-programmable calculator is allowed.

1. (a) Find $L^{-1}\left[\frac{Se^{-\pi s}}{S^2 + 2S + 2}\right]$

5

(b) State true or false with proper justification "There does not exist an analytic function whose real part is $x^3 - 3x^2y - y^3$ ".

5

(c) Prove that $f_1(x) = 1$, $f_2(x) = x$, $f_3(x) = \frac{(3x^2 - 1)}{2}$ are orthogonal over $(-1, 1)$.

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(d) Using Green's theorem in the plane, evaluate $\int_C (x^2 - y)dx + (2y^2 + x)dy$ around the boundary of the region defined by $y = x^2$ and $y = 4$.

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2. (a) Find the fourier cosine integral representation of the function $f(x) = e^{-ax}$, $x > 0$

6

and hence show that $\int_0^{\infty} \frac{\cos ws}{1+w^2} dw = \frac{\pi}{2} e^{-x}$, $x \geq 0$.

(b) Verify laplaces equation for $U = \left(r + \frac{a^2}{r}\right) \cos \theta$ Also find V and f(z).

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(c) Solve the following eqn. by using laplace transform. $\frac{dy}{dt} + 2y + \int_0^t y dt = \sin t$ given that $y(0) = 1$.

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(2083)

2087

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3. (a) Expand $f(x) = \begin{cases} \pi x, & 0 < x < 1 \\ 0, & 1 < x < 2 \end{cases}$ with period 2 into a fourier series. 6
- (b) A vector field is given by $\vec{F} = (x^2 + xy^2)\mathbf{i} + (y^2 + x^2y)\mathbf{j}$ show that \vec{F} is irrotational 6
and find its scalar potential.
- (c) Find the inverse z - transform of - 8
 $f(z) = \frac{z+2}{z^2 - 2z + 1}, |z| > 1$
4. (a) Find the constants 'a' and 'b' so that the surface $ax^2 - byz = (a + 2)x$ will be 6
orthogonal to the surface $4x^2y + z^3 = 4$ at $(1, -1, 2)$
- (b) Given $L(\text{erf } \sqrt{t}) = \frac{1}{S\sqrt{S+1}}$, evaluate $\int_0^{\infty} t e^{-t} \text{erf}(\sqrt{t}) dt$ 6
- (c) Obtain the expansion of $f(x) = x(\pi - x), 0 < x < \pi$ as a half-range cosine series. 8
- Hence show that - (i) $\sum_1^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$
- (ii) $\sum_1^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$
5. (a) If the imaginary part of the analytic function $W=f(z)$ is $V = x^2 - y^2 + \frac{x}{x^2 + y^2}$ find 6
the real part U.
- (b) If $f(k) = 4^k U(K)$ and $g(k) = 5^k U(K)$, then find the z- transform of $f(k) \cdot g(k)$ 6
- (c) Use Gauss's Divergence theorem to evaluate $\iiint_S \vec{N} \cdot \vec{F} ds$ where $\vec{F} = 4x\mathbf{i} + 3y\mathbf{j} - 2z\mathbf{k}$ 8
and S is the surface bounded by $x = 0, y = 0, z = 0$ and $2x + 2y + z = 4$.

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6. (a) Obtain complex form of Fourier series for $f(x) = \cosh 3x + \sinh 3x$ in $(-3, 3)$. 6
- (b) Find the inverse Laplace transform of $\frac{(s-1)^2}{(s^2-2s+5)^2}$ 6
- (c) Find the bilinear transformation under which $1, i, -1$ from the z -plane are mapped onto $0, 1, \infty$ of w -plane. Also show that under this transformation the unit circle in the w -plane is mapped onto a straight line in the z -plane. Write the name of this line. 8
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