

QP Code : 14544

[3 Hours]

[ Total Marks: 80

- N.B. (1) Question no. 1 is compulsory.  
(2) Attempt any three from the remaining.  
(3) Figures to the right indicate full marks.

1. (a) Find the Laplace Transform of  $\sin t \cos 2t \cos ht$ . 5  
(b) Find the Fourier series expansion of  $f(x) = x^2$   $(-\pi, \pi)$  5  
(c) Find the z-transform of  $\left(\frac{1}{3}\right)^{|k|}$  5  
(d) Find the directional derivative of  $4xz^2 + x^2yz$  at  $(1, -2, -1)$  in the direction of  $2\bar{i} - \bar{j} - 2\bar{k}$  5
2. (a) Find an analytic function  $f(z)$  whose real part is  $e^x(x \cos y - y \sin y)$  6  
(b) Find inverse Laplace Transform by using convolution theorem  $\frac{1}{(s-3)(s+4)^2}$  6  
(c) Prove that  $\bar{F} = (6xy^2 - 2z^3)\bar{i} + (6x^2y + 2yz)\bar{j} + (y^2 - 6z^2x)\bar{k}$  is a conservative field. 8  
Find the scalar potential  $\phi$  such that  $\nabla \phi = \bar{F}$ . Hence find the workdone by  $\bar{F}$  in displacing a particle from  $A(1,0,2)$  to  $B(0,1,1)$  along AB.
3. (a) Find the inverse z-transform of  $F(z) = \frac{z^3}{(z-3)(z-2)^2}$  6  
(i)  $2 < |z| < 3$  (ii)  $|z| > 3$   
(b) Find the image of the real axis under the transformation  $w = \frac{2}{z+i}$  6  
(c) Obtain the Fourier series expansion of 8  
 $f(x) = \pi x; 0 \leq x \leq 1$   
 $= \pi(2-x); 1 \leq x \leq 2$   
Here deduce That  $\frac{1}{1^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{8}$
4. (a) Find the Laplace Transform of 6  
 $f(t) = E; 0 \leq t \leq \frac{p}{2}$   
 $= -E; \frac{p}{2} \leq t \leq p, \quad f(t+p) = f(t)$

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(b) Using Green's theorem evaluate  $\int_c \frac{1}{y} dx + \frac{1}{x} dy$  where  $c$  is the boundary of the region bounded by  $x=1$ ,  $x=4$ ,  $y=1$ ,  $y=\sqrt{x}$  6

(c) Find the Fourier integral for  $f(x) = 1-x^2$ ,  $0 \leq x \leq 1$   
 $= 0$   $x > 1$  8

Hence evaluate  $\int_0^{\infty} \frac{\lambda \cos \lambda - \sin \lambda}{\lambda^3} \cos\left(\frac{\lambda}{2}\right) d\lambda$

5. (a) If  $\vec{F} = x^2\vec{i} + (x-y)\vec{j} + (y+z)\vec{k}$  moves a particle from  $A(1, 0, 1)$  to  $B(2, 1, 2)$  along line  $AB$ . Find the work done. 6

(b) Find the complex form of Fourier series  $f(x) = \sin^2 ax$   $(-\ell, \ell)$  6

(c) Solve the differential equation using Laplace Transform.  
 $(D^2+2D+5)y = e^{-t} \sin t$   $y(0) = 0$   $y'(0) = 1$  8

6. (a) If  $\int_0^{\infty} e^{-2t} \sin(t+\alpha) \cos(t-\alpha) dt = \frac{3}{8}$  find the value of  $\alpha$ . 6

(b) Evaluate  $\iint_s (y^2z^2\vec{i} + z^2x^2\vec{j} + z^2y^2\vec{k}) \cdot \vec{n} ds$  where  $s$  is the hemisphere  $x^2+y^2+z^2=1$  above  $xy$ - plane and bounded by this plane. 6

(c) Find Half range sine series for  $f(x) = \ell x - x^2$   $(0, \ell)$  8

Hence prove that  $\frac{1}{1^6} + \frac{1}{3^6} + \dots = \frac{\pi^6}{960}$