

(Time: 3 hours)

Max Marks: 80

Note: (1) Question No. 1 is Compulsory.

(2) Answer any three questions from Q.2 to Q.6.

(3) Figures to the right indicate full marks.

Q1.

a) Solve  $(y^2 e^{xy^2} + 4x^3)dx + (2xye^{xy^2} - 3y^2)dy = 0$  5

b) Solve  $(D^2 - 4D + 4)y = e^{2x} + \cos 2x$  5

c) Show that  $\int_0^\infty \frac{e^{-x^3}}{\sqrt{x}} dx * \int_0^\infty y^4 e^{-y^6} dy = \frac{\pi}{9}$  5

d) Change the order of the following integration 5

$$I = \int_0^1 \int_{\sqrt{2x-x^2}}^{1+\sqrt{1-x^2}} f(x,y) dy dx$$

Q2.

a) Evaluate  $I = \iiint \frac{dx dy dz}{(x^2 + y^2 + z^2)^{3/2}}$ , over the volume V bounded by the spheres  $x^2 + y^2 + z^2 = a^2$  and  $x^2 + y^2 + z^2 = b^2$ ,  $(b > a)$  6

b) Show that the length of the arc of the parabola  $y^2 = 4ax$  from the vertex to the end of the latus rectum is  $a[\sqrt{2} + \log(1 + \sqrt{2})]$ . 6

c) Solve by using method of variation of parameters 8

$$\frac{d^2y}{dx^2} + y = \sec x \tan x$$

Q3.

a) Show that  $\int_0^\pi \frac{\log(1+a\cos x)}{\cos x} dx = \pi \sin^{-1} a, 0 \leq a \leq 1$ . Hence 6  
 evaluate  $\int_0^\pi \frac{\log(1+\cos x)}{\cos x} dx$

b) Evaluate  $I = \iint y^2 dx dy$  over the area outside  $x^2 + y^2 - ax = 0$  and inside  $x^2 + y^2 - 2ax = 0$ . 6

c) Evaluate  $I = \int_0^1 \int_0^{1-x} \int_0^{1-x-y} x^2 y z dx dy dz$  8

Q4.

a) Solve  $\cosh x \frac{dy}{dx} = 2 \cosh^2 x \sinh x - y \sinh x$  6

b) Solve  $\frac{d^2 y}{dx^2} + 3 \frac{dy}{dx} + 2y = e^{e^x}$  6

c) Show that  $\int_0^\infty \frac{dx}{(e^x + e^{-x})^n} = \frac{1}{4} \beta\left(\frac{n}{2}, \frac{n}{2}\right)$ , hence find the value of  $\int_0^\infty \operatorname{sech}^6 x dx$  8

Q5.

a) Evaluate  $I = \int_{-1}^1 \int_0^{1-x} x^{1/3} y^{-1/2} \left((1-x-y)^{1/2}\right) dx dy$  6

b) Find the area inside the circle  $r = a$  and outside the cardioid  $r = a(1 + \cos\theta)$  6

c) Solve  $xy(1 + xy^2) \frac{dy}{dx} = 1$  8

Q6.

a) Solve  $(D^2 + 2)y = x^2 e^{3x} + x \sin 3x$  6

b) Solve  $x e^x (dx - dy) + e^x dx + y e^y dy = 0$  6

c) Change the order of integration and evaluate 8

$$I = \int_0^a \int_0^x \frac{dx dy}{(y+a)\sqrt{(a-x)(x-y)}}$$

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