

(Time: 3 Hours)

Max. Marks: 80

- N.B:** 1) Question No.1 is **COMPULSORY**  
 2) Answer **ANY THREE** questions from **Q. 2 to Q. 6**  
 3) Figures to the right indicate full marks

**Q.1** a) Solve 5

$$(2x - 2x^3y^2)dy + (x^2y^3 + 2y)dx = 0$$

b) Show that 5

$$\int_0^{\pi/2} \sqrt{\sin \theta} d\theta * \int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin \theta}} = \pi$$

c) Evaluate 5

$$\int_0^{\log 2} \int_0^x \int_0^{x-y} e^{x+y+z} dz dy dx$$

d) Solve 5

$$(D^2 + 3D + 2)y = \sin(e^x)$$

**Q.2** a) Evaluate  $\int_0^1 \frac{x^a - 1}{\log x} dx$  using DUIS rule 6

b) Change the order of Integration 6

$$\int_0^a \int_{\sqrt{a^2 - x^2}}^{x+3a} f(x, y) dy dx$$

c) Solve by the method of variation of parameters 8

$$\frac{d^2y}{dx^2} + y = \sec x \tan x$$

**Q.3** a) Evaluate  $\iiint dx dy dz$  over the solid region of paraboloid  $x^2 + y^2 = 4z$  cut off by the plane  $z = 4$  6

b) Solve 6

$$y^4 dx = \left( \frac{1}{x^{3/4}} - xy^3 \right) dy$$

c) Prove that 8

$$\left( \int_0^3 \frac{x^{3/2}}{\sqrt{3-x}} dx \right) * \left( \int_0^1 \frac{1}{\sqrt{1-x^{1/4}}} dx \right) = \frac{432}{35} \pi$$

- Q.4** a) Solve 6  
 $[y \sin(xy) + xy^2 \cos(xy)]dx = -[x \sin(xy) + x^2y \cos(xy)]dy$
- b) Change the order of the integration and evaluate 6  

$$\int_0^1 \int_{-\sqrt{y}}^{-y^2} xy \, dx \, dy$$
- c) Solve 8  
 $(D^2 + 2)y = e^x \cos x + x^2 e^{3x}.$
- Q.5** a) Solve  $x \sin x \, dy + (xy \cos x - y \sin x - 2)dx = 0$  6
- b) Evaluate  $(D^2 + 1)y = 2^x + \sin x \sin 2x$  6
- c) Using Polar co-ordinates, evaluate  $\iint \frac{(x^2+y^2)^2}{x^2y^2} \, dx \, dy$  over the area 8  
 common to the circles  $x^2 + y^2 = ax$  and  $x^2 + y^2 = by$ ,  $a > 0, b > 0$
- Q.6** a) Find the length of the cardioid  $r = a(1 - \cos \theta)$  lying outside the circle 6  
 $r = a \cos \theta$
- b) Evaluate  $\iint xy \, dx \, dy$  over the region bounded by the curves 6  
 $y = 4x, x + y = 3, y = 0$  and  $y = 2$
- c) Evaluate  $\iiint (x^2 + y^2 + z^2) \, dx \, dy \, dz$  over the positive octant of the 8  
 sphere  $x^2 + y^2 + z^2 = 4$
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