University of Mumbai

Examinations Summer 2022

Program No: 1T01831

Examination: F.E. (Sem I) (ALL BRANCHES) (Rev 2019 'C'-Scheme)

Subject (Paper Code): 58651 // Engineering Mathematics - I

Time: 2 hour 30 minutes **Max. Marks: 80**

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DATE: 27/6/2022 QP CODE: 95126

Q I.	**Choose the correct option for following questions. All the Questions are compulsory and carry equal marks.**
1.	The value of $\left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right)^{10}+\left(\frac{1}{\sqrt{2}}-\frac{i}{\sqrt{2}}\right)^{10}$ is equal to
Option A:	$\frac{\pi}{2}$
Option B:	0
Option C:	$\frac{\pi}{3}$
Option D:	$\frac{\pi}{4}$
2.	What is the value of $\log(i)$
Option A:	$i\,\frac{\pi}{2}$
Option B:	0
Option C:	-2
Option D:	$-i\,\frac{\pi}{2}$
3.	$If\ u=\log(tanx+tany)$ then the value of $sin2x\frac{\partial u}{\partial x}+sin2y\frac{\partial u}{\partial y}$ is
Option A:	2
Option B:	-1
Option C:	0
Option D:	- 2
4.	All the stationary points of the function $x^3+3xy^2-15x^2-15y^2+72x$ are
Option A:	$(6,0),\quad (4,0),\quad (5,1),\quad (5,-1)$
Option B:	$(6,4),\quad (4,0),\quad (5,0),\quad (5,1)$
Option C:	$(6,0),\quad (0,0),\quad (5,1),\quad (5,-1)$
Option D:	$(0,0),\quad (4,0),\quad (5,1),\quad (5,-2)$
5.	If $A=\begin{bmatrix}1&0&0\\0&-1&0\\0&0&3\end{bmatrix}$, then rank of A is
Option A:	2
Option B:	3
Option C:	1
Option D:	0

6.	The modulus and principal value of the argument of $\frac{(1+i\sqrt{3})^{13}}{(\sqrt{3}-i)^{11}}$ is
Option A:	$\frac{1}{4}(cos\frac{\pi}{6}+isin\frac{\pi}{6})$
Option B:	$4(cos\frac{\pi}{3}+isin\frac{\pi}{3})$
Option C:	$\frac{1}{4}(cos\frac{\pi}{3}+isin\frac{\pi}{3})$
Option D:	$4(cos\frac{\pi}{6}+isin\frac{\pi}{6})$
7.	The real part of $\cos^{-1}\left(\frac{3i}{4}\right)$ is
Option A:	π
Option B:	2π
Option C:	$-\pi$
Option D:	$\pi/2$
8.	$If\ u=\frac{\sqrt{xy}}{\sqrt{x}+\sqrt{y}}\ \ then\ x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}\ is$
Option A:	$\frac{u}{2}$
Option B:	$\frac{-u}{2}$
Option C:	$2u$
Option D:	$\sqrt{2}\,u$
9.	Stationary point is a point where $f(x,y)$ has
Option A:	$\frac{\partial f}{\partial x}=0$
Option B:	$\frac{\partial f}{\partial y}=0$
Option C:	$\frac{\partial f}{\partial x}=0,\ \ \frac{\partial f}{\partial y}=0$
Option D:	$\frac{\partial f}{\partial x}<0\ \ \frac{\partial f}{\partial x}>0$
10.	For non-singular matrices P and Q, PAQ is in the normal form of a matrix A, then A^{-1} can be found by
Option A:	$A^{-1} = Q^{-1} P$
Option B:	$A^{-1} = P Q^{-1}$
Option C:	$A^{-1} = QP$
Option D:	$A^{-1} = Q P^{-1}$

Q II. (20 Marks)	**Solve any Four out of Six.** **5 marks each**
A	Prove that: $\frac{\sin 6\theta}{\sin 2\theta} = 16cos^2\theta - 16cos^2\theta + 3$
B	Considering only principal values separate into real and imaginary parts $i^{\log(1+i)}$.
C	If z = $tan^{-1}\left(\frac{y}{x}\right)$, find the value of $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}$.
D	Find the extreme value of the function $xy(3 - x - y)$.
E	Express the matrix $\begin{bmatrix} 3i & -1+i & 3-2i \\ 1+i & -i & 1+2i \\ -3-2i & -1+2i & 0 \end{bmatrix}$ as a sum of Hermitian and skew Hermitian matrix.
F	If $y = a\cos(logx) + b\sin(logx)$, then show that $x^2 y_{n+2} + (2n+1)x\, y_{n+1} + (n^2 + 1)y_n = 0$.

Q III. (20 Marks)	**Solve any Four out of Six.** **5 marks each**
A	Find all the values of $(1+i)^{\frac{1}{3}}$ and show that their continued product is $(1+i)$.
B	Separate into real and imaginary parts $tan^{-1}(\alpha + i\beta)$
C	If $u = sin^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$, prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -\frac{\sin u \cos 2u}{4cos^3 u}$.
D	Divide 24 into 3 parts such that the continued product of the first, square of second and cube of the third is maximum using Lagrange's method.
E	Find a, b, c if A is orthogonal matrix where A $=\frac{1}{3}\begin{bmatrix} a & b & c \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$. Hence find inverse of A.
F	Investigate for what values of λ and μ the system of equations $x + y + z = 6$; $x + 2y + 3z = 10$; $x + 2y + \lambda z = \mu$ has (i) no solution (ii) a unique solution (iii) an infinite no. of solutions.

Q IV. (20 Marks)	**Solve any Four out of Six.** **5 marks each**
A	Prove that $(1 + i\sqrt{3})^n + (1 - i\sqrt{3})^n = 2^{n+1}\cos\frac{n\pi}{3}$.
B	Prove that $\sinh^{-1}(\tan\theta) = log\left[\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right]$
C	If $u = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, then show that $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0$
D	Find n[th] derivatives of $\frac{x}{(x-1)(x-2)(x-3)}$.

E	Find non-singular matrices P and Q such that $A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$ is reduced to normal form. Also find its rank.
F	Using De Moivre's theorem prove that $$cos^6\theta - sin^6\theta = \frac{1}{16}(cos6\theta + 15cos2\theta).$$