

Duration: 3 Hours

Max. Marks: 80

N.B. (1) Question No. 1 is COMPULSORY.

(2) Answer ANY THREE questions from Q.2 to Q.6.

(3) Use of Statistical Tables permitted.

(4) Figures to right indicate full marks.

**Que. 1 a.** Evaluate the integral  $\int_C \frac{1}{(z^2+1)(z^2+4)} dz$ , where **5**

C is the circle  $|z-i|=1$ .

**b.** A random variable X has the distribution **5**

X:	0	1	2	3	4	5	6
p(x):	k	3k	5k	7k	9k	11k	13k

Find i) k ii)  $P(3 < X \leq 6)$

**c.** Show that the vectors  $V_1 = (1, 2, 4)$ ,  $V_2 = (2, -1, 3)$  and  $V_3 = (0, 1, 2)$  are linearly independent. **5**

**d.** Find Rank correlation coefficient for the following data **5**

x :	12	17	22	27	32
y :	113	119	117	115	121

**Que. 2 a.** Find usual inner product between two vectors **6**  
 $(2, -3, 1)$  and  $(3, 4, -5)$ . Find norm of each vectors and verify Cauchy Schwarz inequality.

**b.** Find the Extremal of  $\int_{x_1}^{x_2} \sqrt{1+(y')^2} dx$ . **6**

**c.** The following table gives data concerning the savings bank deposits (X) in lakhs and number of strikes and lockouts(Y) over a period of 7 years. Calculate the correlation coefficient and the regression lines. **8**

X	51	54	55	59	65	60	70
Y	38	44	33	36	33	23	10

**Que. 3 a.** Show that the  $V = \{(x, y) | x = 7y\}$  is a subspace of  $R^2$ . **6**

The weekly wages of 1000 workmen are normally distributed around a mean of Rs 70 and standard deviation Rs 5. **6**

**b.** Estimate the number of workers whose weekly wages will be  
 (i) between 69 and 72 (ii) more than 75

**c.** Obtain all possible Taylor and Laurent series expansions about  $z=0$  for the function  $\frac{z}{z^2+3z+2}$  indicating the region of convergence **8**

**Que. 4 a.** By using Cauchy residue theorem, evaluate  $\oint_C \frac{\sin^3 z}{(z-\frac{\pi}{6})^2} dz$  where  $C$  is a circle  $|z|=2$  **6**

**b.** A continuous random variable  $X$  has a probability density function  $f(x) = kx^2 e^{-x}$ ,  $x \geq 0$ . **6**

Find  $k$ , mean and variance.

**c.** Using Rayleigh-Ritz method, find approximate solution for the extremal of  $\int_0^1 (y'^2 - 4y^2 + 2x^2 y) dx$ ,  $y(0)=1$ ,  $y(1)=0$  **8**

**Que. 5 a.** Ten students got the following percentage of marks in mathematics and statistics **6**

	78	36	98	25	75	82	90	62	65	39
Maths										
Stats	84	51	91	60	68	62	86	58	53	47

Calculate the coefficient of correlation.

**b.** Using Gram-Schmidt process, construct an orthonormal basis of  $R^3$  for  $S = \{(3, 0, 4) (-1, 0, 7) (2, 9, 11)\}$  **6**

**c.** Reduce quadratic form **8**

$x_1^2 + 2x_2^2 + 2x_3^2 - 2x_1x_2 + x_1x_3 - 2x_2x_3$  to diagonal form by congruent transformation. Obtain transformation applied in the reduction and Find the rank, index and class value.

- Que. 6**
- a.** Find the curve on which the functional  $\int_0^1 \{y'^2 + 12xy\} dx$  with  $y(0)=0, y(1)=1$  is extremal. **6**
  - b.** A car hire firm has two cars, which they hires out day by day. The number of demands for a car on each day is distributed as a Poisson variate with mean 1.5. Calculate the proportion of days on which (i) neither car is used (ii) some demands are refused. **6**
  - c.** Find a singular value decomposition of the matrix  $\begin{bmatrix} 4 & 0 \\ 3 & -5 \end{bmatrix}$  **8**

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