University of Mumbai   **QP-3**

Examinations Summer

Program No : 1T01034

Name of the Examination : S.E.(Electronics and Telecommunication)(SEM-IV)(Choice Base
Credit Grading System) (R- 19) (C Scheme)

Subject (Paper Code): 40821 / / Engineering Mathematics-IV

Time: 2 hour 30 minutes   Max. Marks: 80

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S.E.(Electronics and Telecommunication)(SEM-IV)(Choice Base Credit Grading System) (R- 19) (C Scheme) / 40821 -
Engineering Mathematics-IV   DATE: 17/5/2022   QP CODE: 90654

NB:

1. All the questions are COMPULSORY.

2. Write the correct option for multiple choice question (MCQ) in Q. 1

3. Q.2 to Q. 4 have internal choice within question.

4. Figures to the right indicate full marks.

5. Use of scientific calculator is allowed.

Q1. (20 Marks)	Choose the correct option for following questions. All the Questions are compulsory and carry equal marks **2marks each**
1.	The function $f(z) = \dfrac{2}{(z+5)^3(z-2)^4}$ has poles at z=-5 of order …. and z=2 of order …
Option A:	3, 2
Option B:	3, 4
Option C:	2, 4
Option D:	4, 5
2.	If f(z) is analytic function and f'(z) is continuous at all points inside and on simple closed curve 'C' , then
Option A:	$\displaystyle\oint_C f(z)dz = 0$
Option B:	$\displaystyle\oint_C f(z)dz \neq 0$

Option C:	$$\oint_C f(z)dz = 2\pi i f(a)$$
Option D:	$$\oint_C f(z)dz = 1$$
3.	The rank of the matrix A is the
Option A:	Dimension of the row space A
Option B:	Dimension of the column space A
Option C:	Both option A and B
Option D:	Dimension of the null space of A and B
4.	Let $Q(X) = X^T A X$, be a quadratic form in 'n' variables then which of the following statement is wrong
Option A:	The total number of non-zero terms in the canonical form of quadratic form is called as rank of quadratic form
Option B:	The number of positive square terms in the canonical form is called as Index of the quadratic form
Option C:	The difference between number of positive and negative terms in the canonical form is called as signature of the quadratic form
Option D:	Signature of the quadratic form is greater than rank of quadratic form
5.	The necessary condition for $\int_{x_1}^{x_2} f(x, y, y')\, dx$ to be maximum or minimum is
Option A:	$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$
Option B:	$\frac{\partial f}{\partial y} + \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$
Option C:	$\frac{\partial f}{\partial y'} - \frac{d}{dx}\left(\frac{\partial f}{\partial y}\right) = 0$
Option D:	$\frac{\partial f}{\partial y'} + \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$
6.	If the vectors [k, k, -2] and [k, -2, 12] are orthogonal vectors, then the values of 'k' are

Option A:	k = 6 or k = −4
Option B:	k = 6 or k = 4
Option C:	k = 3 or k = −4
Option D:	k = 2 or k = −2
7.	The value of coefficient of correlation lies between
Option A:	0 to 1
Option B:	-∞ to 1
Option C:	0 to ∞
Option D:	-1 to 1

8.	The rank correlation coefficients of the following data is

X	23	25	27	29	31	33
Y	43	45	47	49	51	53

Option A:	0
Option B:	-1
Option C:	1
Option D:	0.99
9.	If 'X' and 'Y' are two normal variables with mean 40 and 50 with standard deviation 4 and 3 respectively, what is the distribution of X+Y
Option A:	N(90, 7)
Option B:	N(90 , 3)
Option C:	N(90, 5)
Option D:	N(90,4)
10.	What would be the expectation of the number of failures preceding the first success in an infinite series of independent trials with the constant probability of success p and failure q
Option A:	$\frac{p}{q}$
Option B:	$\frac{q}{p}$
Option C:	$\frac{p+1}{q}$
Option D:	$\frac{p^2}{q^2}$

Q2 (20 Marks)	**Solve any Four out of Six** **5 marks each**
A	Find the extremals of the functional $\int_0^1 \{y'^2 + 12xy\}\, dx$ subject to $y(0) = 0$ and $y(1) = 1$
B	For real values of a, b and θ, using Cauchy- Schwarz inequality, show that $(a\cos\theta + b\sin\theta)^2 \leq a^2 + b^2$
C	Evaluate $\oint_C \dfrac{\sin^6 z}{\left(z-\frac{\pi}{6}\right)^3}\, dz$ where C is the circle |z|=1
D	Find the probability that at most 4 defective bulbs will be found in a box of 200 bulbs, if it is known that 25 of the bulbs are defective.
E	Ten students got the following percentage of marks in mathematics and statistics

Maths	78	36	98	25	75	82	90	62	65	39
Stats	84	51	91	60	68	62	86	58	53	47

Calculate the coefficient of correlation.

F	A bolt is manufactured by three machines A, B, and C . A turns out twice as many times as B, and machines B and C produce equal number of items . 3% of bolts produced by A and B are defective and 5% of bolts produced by C are defective . All bolts are put into one stock pile and one is chosen from this pile. What is the probability that it is defective?

Q. 3 (20 Marks)	**Solve any Four out of Six** **5 marks each**
A	Test for an extremal of the functional $\int_0^1 \{xy + y^2 - 2y^2 y'\}\, dx$ with y(0)=0, y(1)=2
B	Show that the set $W = \{[x,\ y,\ z]\mid y = x + z\}$ is a subspace of R^3 under usual addition and scalar multiplication.
C	Evaluate the complex line Integral $\displaystyle\int_0^{1+i} (x - y + ix^2)\,dz$ along (a) the straight line from z=0 to z=1+i (b) the real axis from z=0 to z=1 & then along a line parallel to the

	imaginary axis from z=1 to z=1+i
D	Reduce the matrix of the quadratic form $6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 + 4x_1x_3 - 2x_2x_3$ to canonical form by congruent transformation and find rank, signature, value class.
E	Obtain the equations of the lines of regression for the following data.
F	A random variable X has the following probability distribution

E table:

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

F table:

X	-2	-1	0	1	2	3
P	0.1	K	0.1	2K	0.2	3K

(i) Find the constant K.   (ii) Find the mean and variance of X.

Q. 4 **(20 Marks)**	**Solve any Four out of Six** **5 marks each**
A	Using Rayleigh-Ritz method, find an approximate solution for the extremal of the functional $\int_0^1 \left\{ xy + \frac{1}{2}y'^2 \right\} dx$ subject to y(0)=y(1)=0
B	Using Gram-Schmidt process, construct an orthonormal basis for the plane x+y+z=0
C	Obtain Taylor's and Laurent's series expansions of $f(z) = \frac{2}{(z-1)(z-2)}$ When 1. |z|<1 2. 1<|z|<2 3. |z|>2
D	Find Singular value decomposition of $\begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}$
E	Fit a straight line of the form y=a+bx to the following data
F	A random variable x has probability density function $f(x) = \begin{cases} kx^2 e^{-x} & x > 0, \quad k > 0 \\ 0 & Otherwise \end{cases}$ Find 'k' and hence find mean and variance.

E table:

X	1	3	5	7	8	10
Y	8	12	15	17	18	20