University of Mumbai

Examinations Summer 2022

S.E.(Electronics and Telecommunication)(SEM-III)
(Choice Base Credit Grading System) (R- 19) (C Scheme)
Paper Code 51221 / / Engineering Mathematics-III

Time: 2 hour 30 minutes Max. Marks: 80

DATE: 3/6/022 QP CODE:92338

Q1.	**Choose the correct option for following questions. All the Questions are compulsory and carry equal marks**
1.	If $L\{f(t)\} = \frac{2}{s^3}e^{-s}$ then $L\{f(2t)\}$
Option A:	$\frac{8}{s^3}e^{-\frac{s}{2}}$
Option B:	$\frac{6}{s^3}e^{-\frac{s}{2}}$
Option C:	$\frac{8}{s^3}e^{-s}$
Option D:	$\frac{8}{s^3}e^{-1}$
2.	The Fourier Coefficient b_n of $f(x) = 4 - x^2$ in $(0,2)$ is
Option A:	$\frac{2}{n\pi}$
Option B:	$\frac{4}{n\pi}$
Option C:	$\frac{8}{n^2\pi^2}$
Option D:	$\frac{16}{n^2\pi}$
3.	The directional derivative of $\Phi = x^2 + 3y^2 + 2z^2$ in the direction of the vector $2\hat{\imath} - \hat{\jmath} - 2\hat{k}$ at the point$(1, -3, 2)$ is given
Option A:	2
Option B:	4
Option C:	-2
Option D:	-4
4.	The Maximum direction derivative of $\Phi = (4x - y + 2z)^2$ at $(1,2,1)$ is
Option A:	$8\sqrt{11}$
Option B:	$\sqrt{7}$
Option C:	5
Option D:	$8\sqrt{21}$

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5.	If the matrix $A = \begin{bmatrix} a & 1 \\ 2 & 3 \end{bmatrix}$ has 1 as an eigen value, then trace A is
Option A:	4
Option B:	5
Option C:	6
Option D:	7
6.	The characteristic polynomial of matrix $A = \begin{bmatrix} 0 & 0 & -c \\ 1 & 0 & -b \\ 0 & 1 & -a \end{bmatrix}$ is
Option A:	$x^3 + ax^2 + bx + c$
Option B:	$(x-a)(x-b)$
Option C:	$(x-1)(x-abc)^2$
Option D:	$(x-1)^2(x-abc)$
7.	If $v = \tan^{-1}\left(\frac{y}{x}\right)$ is the imaginary part of the analytic function $f(z)$, then
Option A:	$f(z) = \log(z) + c$
Option B:	$f(z) = \frac{1}{2}\log(z) + c$
Option C:	$f(z) = \log(z - \sin(z)) + c$
Option D:	$f(z) = i\log(z) + z^2\tan(z) + c$
8.	If $\bar{r} = x\,\hat{\imath} + y\hat{\jmath} + z\hat{k}$ then $div(\bar{r}) =$ ____
Option A:	2
Option B:	3
Option C:	5
Option D:	6
9.	If $f(x) = \begin{cases} 0 & , -\frac{\pi}{\omega} < x < 0 \\ E\sin(\omega x) & , 0 < x < \frac{\pi}{\omega} \end{cases}$ .with period $\frac{2\pi}{\omega}$, where E and ω are non zero constants, then the value of a_1 in Fourier Series expansion of $f(x)$ is
Option A:	0
Option B:	1
Option C:	-1
Option D:	2
10.	Let $\bar{F} = (x-y-z)\hat{\imath} + (y-z-x)\hat{\jmath} + (z-x-y)\hat{k}$ and S is a paraboloid $x^2 + y^2 = 4 - z$, $z \geq 0$. Then by Stoke's Theorem the value of $I = \int_C \bar{F} \cdot dr$
Option A:	1
Option B:	2
Option C:	0
Option D:	3

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Q2, (20 Marks)	Solve any Four out of Six 5 marks each
A	Find the Fourier Series of $f(x) = x^3$ in the interval$(-\pi, \pi)$
B	Find the values of a, b, c if $\bar{F} = (axy + bz^3)i + (3x^2 - cz)j + (3xz^2 - y)k$ is irrotational.
C	Find the value of α if $\int_0^\infty e^{-2t} \sin(t+\alpha)\cos(t-\alpha)\,dt = \frac{1}{4}$
D	Find $L^{-1}\left\{s \log\left(\frac{s+1}{s-1}\right)\right\}$
E	Find the Eigen values and Eigen vectors of $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$
F	Show that $v = e^x(x\sin y + y\cos y)$ is harmonic and also find its corresponding analytic fuction.

Q3. (20 Marks)	Solve any Four out of Six 5 marks each
A	Find $L\left\{\frac{sin^2 2t}{t}\right\}$
B	Find $L^{-1}\left\{\frac{s}{(s^2+4)(s^2+1)}\right\}$
C	Find the half range cosine series $f(x) = lx - x^2$, on $0 < x < l$.
D	Show that $\bar{F} = (ye^{xy}\cos z)\hat{\imath} + (xe^{xy}\cos z)\hat{\jmath} - (e^{xy}\sin z)\hat{k}$ is irrotational and also find the scalar potential Φ such that $\bar{F} = \nabla\Phi$
E	If $A = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$, then prove that $3tanA = Atan3$.
F	Find the orthogonal trajectories of the family of curves given by $e^{-x}\cos y + xy = \alpha$.

Q4. (20 Marks)	Solve any Four out of Six 5 marks each
A	Find the analytic function $f(z) = u + iv$ in terms of z if $u + v = \frac{\sin(2x)}{\cosh(2y) - \cos(2x)}$
B	Evaluate using Laplace transform, $\int_0^\infty e^{-t}\left(\int_0^t \frac{sinu}{u}du\right)dt$
C	Find the Fourier Series expansion $f(x) = x^2$ in $(0, 2\pi)$
D	Using Cayley Hamilton theorem for matrix $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, express $2A^5 - 3A^4 + A^2 - 4I$ as a linear polynomial in A.
E	Using Green's theorem evaluate $\oint_C x^2(1-y)dx + (2y+1)dy$ Where C is the circle $x^2 + y^2 = a^2$
F	Find $L^{-1}\left\{\frac{5s^2+8s-1}{(s+3)(s^2+1)}\right\}$

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