

Duration: 3hrs

[Max Marks: 80]

- N.B. : (1) Question No 1 is Compulsory.
 (2) Attempt any three questions out of the remaining five.
 (3) All questions carry equal marks.
 (4) Assume suitable data, if required and state it clearly.

1 Attempt any FOUR [20]

- a If R be a relation in the set of integers z defined by
 $R = \{ (x, y) : x \in z, y \in z, x - y \text{ is divisible by } 3 \}$
 Show that the relation R is an equivalence relation.
- b Prove using Mathematical Induction that
 $P(n) = 1.1! + 2.2! + \dots + 3.3! = (n + 1)! - 1$
- c Design an FSM in which input is valid if it ends in "1011" over $\Sigma = \{0,1\}$
- d Design NFA for the regular expression

$$R = (0(0 + 1)^*10)$$

e Differentiate between DFA and NFA

2 a Define Poset. Draw Hasse diagram which represents the partial order relation. [10]

$$R = \{ (a, b) | a \text{ divides } b \} \text{ on } \{1,2,3,4,6,8,12\}$$

b Convert the following NFA to DFA: [10]

Q/ Σ	0	1
$\rightarrow p$	p,q	p
q	r,s	t
r	p,r	t
s^*	ϕ	ϕ
t^*	ϕ	ϕ

3 a Simplify the following CFG: [10]

$$S \rightarrow aAa | bBb | BB$$

$$A \rightarrow C$$

$$B \rightarrow A | S$$

$$C \rightarrow S | \epsilon$$

b Write a short note on Warshall's algorithm. [10]

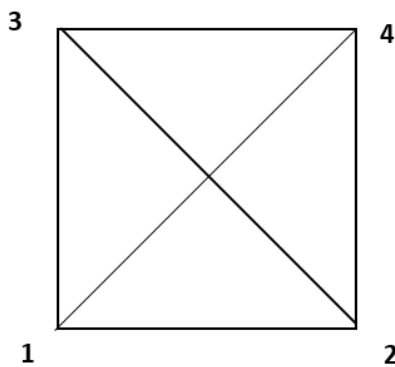
Let $A = \{a_1, a_2, a_3\}$ and R be a relation on A whose matrix is:

$$M_r = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}. \text{ Find transitive closure of R using Warshall's algorithm.}$$

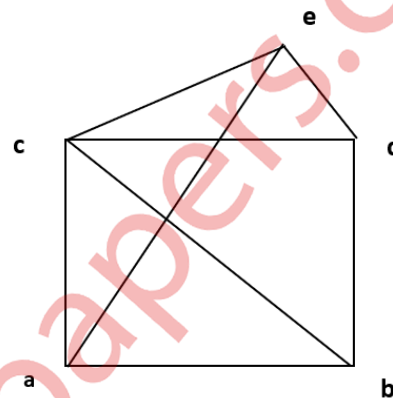
- 4 a Design PDA to check odd palindrome over $\Sigma = \{0,1\}$ [10]
 b Give and explain formal definition of pumping lemma for regular language and prove that the following language is not regular. [10]

$$L = \{ a^m b^{m-1} \mid m > 0 \}$$

- 5 a Design Moore machine for the following: If input ends in '101' then output should be 'A', if input ends in '101' output should be 'B', otherwise output should be 'C' and convert it into Mealy machine. [10]
 b Design a finite automaton to check divisibility by 3 to binary number. [10]
 6 a Determine if the following graphs G1 and G2 are isomorphic or not. [10]



G1



G2

- b Define injective, surjective and bijective functions. If $f: R \rightarrow R$ and $g: R \rightarrow R$ are defined by the formulas: $f(x) = x + 2$ and $g(x) = x^2$. Find [10]
1. $f \cdot g \cdot f$
 2. $g \cdot f \cdot g$