

Time: 3 hour

Max. Marks: 80

- Note:** 1. Question no. 1 is compulsory.  
2. Attempt any **three** questions out of remaining **five** questions.  
3. Figures to the right indicate full marks.

Q1 (a) Find  $L\left[\frac{(\cos at - \cos bt)}{t}\right]$ , (05)

(b) Find the constants k, if  $f(z) = r^3 \cos k\theta + ir^k \sin 3\theta$  is analytic. (05)

(c) If  $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$  Find  $A^{50}$  (05)

(d) If the vector  $\vec{F} = (x + 2y + az)i + (bx - 3y - z)j + (4x + cy + 2z)k$  is irrotational; find the constants a, b, c. (05)

Q2 (a) Find the analytic function  $f(z)$  in terms of  $z$  whose real part is  $u = \sin x \cosh y$  (06)

(b) Obtain the Fourier series for  $f(x) = e^{ax}$  in  $(0, 2\pi)$  (06)

(c) (i) If  $L\{f(t)\} = \frac{1}{s\sqrt{s+1}}$ , then find  $L\{f(2t)\}$   
(ii) Find  $L(t^5 \cosh t)$  (08)

Q3 (a) Find  $L^{-1}\left[\frac{s}{(s^2+4)(s^2+1)}\right]$  by convolution theorem. (06)

(b) Find Fourier expansion of  $f(x) = 2x - x^2$  in  $(0, 3)$  (06)

(c) Evaluate by using Green's theorem  $\int_C (x^2 - y)dx + (2y^2 + x)dy$ , where C is the closed region bounded by  $y = 4$  and  $y = x^2$  (08)

Q4 (a) If  $v = 3x^2y + 6xy - y^3$  show that V is Harmonic function. (06)

(b) Find the Eigenvalues of matrix  $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$  and Show that matrix satisfies the characteristic equation. (06)

(c) Evaluate (i)  $L^{-1} \left\{ \frac{1}{s} \tan^{-1} \frac{1}{s} \right\}$  (ii)  $L^{-1} \left\{ \frac{1}{(s+1)^2+1} \right\}$  (08)

Q5 (a) Obtain the half range Fourier cosine series expansion for

$$f(x) = x(2 - x) \text{ in } (0,2). \quad (06)$$

(b) Find Eigen value and Eigen Vector Of Matrix  $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$  (06)

(c) Show that  $\vec{F} = (y^2 \cos x + z^3)i + (2y \sin x - 4)j + (3xz^2 + 2)k$  is conservative Field. Find (i) Scalar potential for  $\vec{F}$  (ii) the work done in moving an object in this field From  $(0,1,-1)$  to  $(\frac{\pi}{2}, -1, 2)$  (08)

Q6 (a) Find the orthogonal trajectory of family of curves given by

$$2x - x^3 + 3xy^2 = a \quad (06)$$

(b) Evaluate  $\int_0^{\infty} e^{-3t} t \sin t dt$  (06)

(c) Show that the Matrix  $\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$  is diagonalisable. Find the diagonal form D And diagonalizing matrix M. (08)