

Duration: 3 hours

Marks: 80

N.B. (1) Question No. 1 is **COMPULSORY**.

(2) Answer **ANY THREE** questions from Q.2 to Q.6.

(3) Use of Statistical Tables permitted.

(4) Figures to right indicate full marks.

Que. 1 a. Show that the following quadratic form $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4zx$ is **5**
 positive definite

b. Find the extremal of $\int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$ **5**

c. Find a unit vector orthogonal to the vectors (1,1,1) and (0,1,1). **5**

d. Evaluate $\int_0^{1+i} \bar{z} dz$ along the real axis from $z = 0$ to $z = 1$ **5**
 then vertically to $1 + i$.

Que. 2 a. Find the extremal of $\int_0^{\pi} (y'^2 - y^2 + 2xy) dy$ with $y(0) = 0$ **6**
 $y\left(\frac{\pi}{2}\right) = 0$.

b. In a normal distribution 30% of the items are below 35 and 10% of the items are above 60. Find the mean and standard deviation. **6**

c. The lines of regression are $20x - 9y - 107 = 0$ and $15x = 8y + 130$, $V(x) = 16$ Find the means, r , and $V(y)$. **8**

Que. 3 a. In sampling a large number of parts manufactured by a machine the mean number of defectives in a sample of 20 is 2. Out of 1000 such a sample how many would you expect to contain i) 3 defectives ii) less than 3 defectives. **6**

b. Find the line of regression of Y on X for the following data **6**

X	5	6	7	8	9	10	11
Y	11	14	14	15	12	17	16

c. Show that $V = \{(x,0) | x \text{ is real}\}$ with the operations of addition and scalar multiplication defined as $(x_1,0) + (x_2,0) = (x_1+x_2,0)$ and $k(x,0) = (kx,0)$ is a vector space. **8**

- Que. 4 a. Reduce the following quadratic form $3x^2 + 5y^2 + 3z^2 - 2xy - 2yz + 2zx$ to canonical form also find rank signature and index. **6**
- b. Verify Cauchy-Schwartz inequality for $U = (2, 4, -3, 5)$ and $V = (3, 2, 3, -1)$. **6**
- c. Find all possible expansions of $f(z) = \frac{7z-2}{z(z-2)(z+1)}$ about $z = -1$. **8**

- Que. 5 a. If the probability mass function of a random variable is $f(x) = kx(1-x)$, $0 \leq x \leq 1$. find its mean and variance. **6**
- b. Find the orthonormal basis by Gram-Schmidt process to $(-1, 1, 0)$, $(0, 1, 1)$, $(1, 0, 1)$. **6**
- c. Fit a second-degree parabolic curve to the following data **8**

X	1	2	3	4	5	6	7	8	9
Y	2	6	7	8	10	11	11	10	9

- Que. 6 a. Show that $\{(a, 0, 0) \mid a \text{ is real}\}$ is subspace of \mathbb{R}^3 . **6**
- b. Evaluate $\oint_C \frac{(z-3)}{z^2 + 2z + 5} dz$ where C is the circle (a) $|z|=1$ **6**
 (b) $|z+1-i|=2$ (c) $|z+1+i|=2$
- c. Use Rayleigh-Ritz Method to find the extremal of $\int_0^1 (xy + \frac{1}{2}y'^2) dx$ given $y(0)=1$ & $y(1)=0$. **8**
