SE Comp III c-Scheme

40 + 33 + 11

(94)

21 · 11 · 23

(Time: 3 Hours)   Max. Marks: 80

N.B. (1) Question No. 1 is compulsory.
     (2) Answer any three questions from Q.2 to Q.6.
     (3) Use of Statistical Tables permitted.
     (4) Figures to the right indicate full marks

Q1.

(a) Find the Laplace transform of $\frac{\cos 2t \sin t}{e^t}$ [5]

(b) Find k such that $f(z) = \frac{1}{2} \log(x^2 + y^2) + i \tan^{-1}\frac{kx}{y}$ is analytic   [5]

(c) Calculate the Spearman's rank correlation coefficient R   [5]

  X : 10, 12, 18, 18, 15, 40.
  Y : 12, 18, 25, 25, 50, 25.

(d) Find the inverse Laplace transform of $\log\left(\frac{s^2 + a^2}{s^2 + b^2}\right)$.   [5]

Q2.

(a) A continuous random variable has probability density function

$$f(x) = k(x - x^2), \qquad 0 \le x \le 1.$$
$$f(x) = 0 \qquad\qquad\qquad \text{otherwise}$$

  Find k, mean and variance.   [6]

(b) Find the Laplace transform of $e^{-3t} \int_0^t u \sin 3u \, du$.   [6]

(c) Obtain the Fourier series to represent $f(x) = x^2$ in $(0, 2\pi)$

  Hence show that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \ldots\ldots$ [8]

Q3.

(a) If the imaginary part of the analytic function $w = u + iv = f(z)$ is   [6]
  $V = x^2 - y^2 + \frac{x}{x^2 + y^2}$, then show that $u = -2xy + \frac{y}{x^2 + y^2}$.

(b) Find inverse Laplace transform of $\frac{2s^2 - 6s + 5}{(s^3 - 6s^2 + 11s - 6)}$ [6]

(c) Fit a second-degree parabolic curve and estimate y when $x = 10$

  x   : 1, 2, 3, 4, 5, 6, 7, 8, 9,
  y   : 2, 6, 7, 8, 10, 11, 11, 10, 9.   [8]

Q4.

(a) Obtain the Fourier series to represent $f(x) = x^3$ in $(-\pi, \pi)$.   [6]

(b) Find (i) the equation of the lines of Regression (ii) coefficient of correlation for the following data

  X : 65, 66, 67, 67, 68, 69, 70, 72.
  Y : 67, 68, 65, 66, 72, 72, 69, 71.   [6]

(c) Prove that $\int_0^\infty e^{-\sqrt{2}t} \frac{\sin t \sin ht}{t} dt = \frac{\pi}{8}$.   [8]

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Q5.

(a) Find the orthogonal trajectories of the family of curves $x^3 y - xy^3 = c$. [6]

(b) Find the moment generating function of the distribution

X	:	-2	3	1
P(X = x)	:	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{6}$

hence find first four central moments . [6]

(c) Obtain the half range cosine series of $f(x) = x$ in $(0, 2)$

Hence show that $\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4}$ …….. [8]

Q6.(a) Using convolution theorem Find the inverse Laplace transform of $\left[\frac{s^2}{(s^2+2^2)^2}\right]$ [6]

(b) The probability density function of a random variable X is

X	:	1	2	3	4	5	6	7
P(X =x)	:	k	2k	3k	k^2	$k^2 + k$	$2k^2$	$4k^2$

Find k , p(X < 5), P(X > 5) [6]

(c) If $v = 3x^2 y + 6xy - y^3$, show that v is harmonic function
And find the corresponding analytic function . [8]

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