University of Mumbai
Examination Summer 2022
Program: Computer Engineering
Curriculum Scheme: Rev2019
Examination: SE Semester III
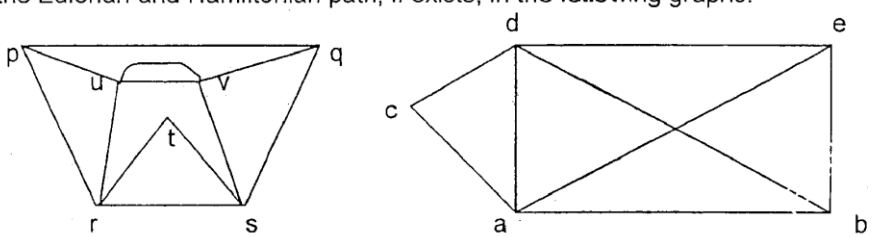Course Code: CSC302 and Course Name: Discrete Structures & Graph Theory
Time: 2 hours 30 minutes   Max. Marks: 80

Q1.	Choose the correct option for following questions. All the Questions are compulsory and carry equal marks
1.	Let the set A is {1, 2, 3} and B is {2, 3, 4}. Then the set A – B is
Option A:	{1, -4}
Option B:	{1, 2, 3}
Option C:	{1}
Option D:	{2, 3}
2.	Let R be a relation on the set A of positive integers. Determine the property of relation R , if (x , y) $\in$ R where R={(x,y) | xy $\geq$ 1 }
Option A:	Anti symmetric
Option B:	Transitive
Option C:	Symmetric
Option D:	Equivalence relation
3.	The statement ($\sim$Q $\leftrightarrow$ R) $\wedge$ $\sim$R is true when?
Option A:	Q: True R: False
Option B:	Q:True R:True
Option C:	Q: False R:True
Option D:	Q: False R: False
4.	How many two-digit numbers can be made from the digits 1 to 9 if repetition is allowed?
Option A:	9
Option B:	18
Option C:	81
Option D:	99
5.	Let P (x) denote the statement "x >5." Which of these have truth value true?
Option A:	P (0)
Option B:	P (1)
Option C:	P (2)
Option D:	**P (9)**
6.	How many binary relations are there on a set S with 5 distinct elements?
Option A:	2^5
Option B:	$\mathbf{2^{25}}$

Option C:	2^{10}
Option D:	2^{15}
7.	The inverse of function $f(x) = x^3 + 2$ is _____
Option A:	$f^{-1}(y) = (y - 2)^{1/2}$
Option B:	$f^{-1}(y) = (y)^{1/3}$
Option C:	$f^{-1}(y) = (y - 2)^{1/3}$
Option D:	$f^{-1}(y) = (y - 2)$
8.	When is a graph said to be bipartite?
Option A:	If it can be divided into two independent sets A and B such that each edge connects a vertex from to A to B
Option B:	If the graph is disconnected
Option C:	If the graph has at least n/2 vertices whose degree is greater than n/2
Option D:	If the graph is connected and it has odd number of vertices
9.	An algebraic structure _____ is called a semigroup.
Option A:	(Q, +, *)
Option B:	(P, *)
Option C:	(P, *, +)
Option D:	(+, *)
10.	Condition for monoid is _____
Option A:	(a+e)=a
Option B:	(a*e)=(a+e)
Option C:	a=(a*(a+e)
Option D:	(a*e)=(e*a)=a

Q2 (20 Marks Each)	
A	**Solve any Two** **5 marks each**
i.	Prove that $8^n - 3^n$ is a multiple of 5 by mathematical induction, $n \geq 1$
ii.	What is a distributed lattice? Draw the hasse diagram of D_{1001}. Whether it is a distributive lattice? Find the inverses of all elements of D_{1001}.
iii.	Determine the Eulerian and Hamiltonian path, if exists, in the following graphs:
B	**Solve any One** **10 marks each**

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i.	What is a transitive closure? Find the transitive closure of R using Warshall's algorithm where A= {a, b, c, d, e, f} & R= {(a, b), (b, c), (c, e), (e, f), (e, b)}
ii.	Let $f(x) = x + 2$, $g(x) = x - 2$ and $h(x) = 3x$ for all $x \in R$. (R is the set of real number). Find i) $f \circ g \circ h$ ii) $h \circ g \circ f$ iii) $f \circ f \circ f$

Q3 (20 Marks Each)	
A	**Solve any Two** **5 marks each**
i.	Let R be the following equivalence relation on the set A = {1, 2, 3, 4, 5, 6}: R = {(1, 1), (1,5), (2, 2), (2,3), (2, 6), (3,2), (3,3), (3,6), (4,4), (5,1), (5,5), (6,2), (6,3), (6,6)} Find the partitions of A induced by R, i.e., find the equivalence classes of R.
ii.	Find truth table for the following expression & determine whether it is a tautology: $(\sim P \land (Q \land R)) \lor (Q \land R) \lor (P \land R) \leftrightarrow R$
iii.	In an auditorium, the chairs are to be numbered with an alphabet followed by a positive integer not exceeding 60. Find the maximum no. of chairs that can be placed in the auditorium.
B	**Solve any One** **10 marks each**
i.	Let $(x1 \land x2) \lor (x1 \land x3) \lor (x2 \land x3)$ be the Boolean expression. Write E (x1, x2, x3) in a Disjunctive & Conjunctive Normal Form.
ii.	Define minimum hamming distance. Find the code words generated by the parity check matrix H given below. $$H = \begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Q4 (20 Marks Each)	
A	**Solve any Two** **5 marks each**
i.	If 5 points are taken in a square of side 2 units, show that at least 2 of them are no more than √2 units apart.

ii.	Consider (3,8) encoding function e : $B^3 \to B^8$ defined by
	    e(000)= 00000000 e(100)= 10I00100
	
	    e(00l)= 10111000 e(101)= 10001001
	
	    e(010)= 00101101 e(110) =00011100

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	e(011)= 10010101 e(111) = 00110001
	and let d be the (8,3) maximum likelihood decoding function associated with e. How many errors can (e, d) correct?

iii.	Find the generating functions for the following sequences:
	    a. 0, 0, 0, 1, 2, 3, 4, 5, 6, 7,…………………………
	    b. 6, -6, 6, -6, 6, -6,…………………………………….

B	**Solve any One** **10 marks each**
i.	Define the term bijective function. Show that the mapping f: R→R given by i) $f(x)= 4x-3$ & ii) $f(x)= 4x+7$ is bijective.

ii.	Explain the following terms with suitable example:
	    a) Incidence matrix
	    b) Hamiltonian path
	    c) Partition set
	    d) Principle of inclusion & exclusion
	    e) Commutative ring