SE CIVIL / SEM-IV / EM-IV / 17.05.2022
QP Code - 92122 C-Scheme

Course Code: PEC401 Course Name : Engineering Mathematics-4
Time: 2 hour 30 minutes Max. Marks: 80

Q1.	Choose the correct option for following questions. All the Questions are compulsory and carry equal marks
1.	Find the angle between the normals to the surface xy = z^2at the points (1,4,2) and (−3,−3,3).
Option A:	$sec^{-1}(\frac{1}{\sqrt{22}})$
Option B:	$cos^{-1}(\frac{1}{\sqrt{22}})$
Option C:	$sec^{-1}(\frac{1}{\sqrt{2}})$
Option D:	$cos^{-1}(\frac{1}{\sqrt{2}})$
2.	Using Stoke's theorem, $\int_C \bar{F}.\overline{dr}$ where $\bar{F} = yzi + xzj + xyk$ and C is the boundary of the circle $x^2 + y^2 + z^2 = 1, z = 0$ is
Option A:	-13
Option B:	33
Option C:	13
Option D:	0
3.	If correlation coefficient, $r = 0.6$ then $b_{yx} = 1.2$ then b_{xy}=?
Option A:	0.45
Option B:	0.2
Option C:	0.72
Option D:	0.3
4.	If two variables oppose each other then the correlation will be
Option A:	Positive correlation
Option B:	Negative correlation
Option C:	Perfect correlation
Option D:	No correlation
5.	In a Poisson distribution if $P(X = 2) = P(X = 3)$ then $P(X = 5)$ is
Option A:	0.84125
Option B:	0.084125

Option C:	0.37256
Option D:	0.037256
6.	For a probability density function of a continuous random variable, the probability of a single point is
Option A:	1
Option B:	2
Option C:	0
Option D:	constant
7.	Which of the following tests would be used to test the mean of a continuous random variable to a population mean?
Option A:	One-sample *t*-test
Option B:	Independent-samples *t*-test
Option C:	Chi-squared *t*-test
Option D:	Dependent-samples *t*-test
8.	Which of the following is not true for a normal distribution?
Option A:	It is a symmetrical distribution.
Option B:	The mean is always zero.
Option C:	The mean, median, mode are always equal.
Option D:	It is a bell-shaped distribution.
9.	The value of $\int_c \frac{\sin z\, dz}{z^6}$, where c is the circle $\lvert z\rvert = 1$ is
Option A:	$\frac{2\pi i}{25}$
Option B:	$\frac{\pi i}{60}$
Option C:	$\frac{3\pi i}{20}$
Option D:	$\frac{5\pi i}{12}$
10.	The value of integral $\oint_c \frac{1}{z-1}\, dz,$ where c is $\lvert z-1\rvert = 2$ is
Option A:	0
Option B:	1
Option C:	-2πi
Option D:	2πi

Q2	**Solve any Four out of Six** **5 marks each**
A	Obtain Laurent's expansion of $f(z) = \frac{z-1}{z^2-2z-3}$ in (i) $1 < \lvert z\rvert < 3$ (ii)

	$|z| > 3$
B	The following results of ranks of were recorded for 11 students. Find Spearman's rank correlation coefficient between the ranks obtained.

Pre-module	Post-module
18	22
21	25
16	17
22	24
19	16
24	29
17	20
21	23
23	19
18	20
14	15

C	A person draws 3 balls from a bag containing 7 blue, 5 yellow, 3 purple balls. He is offered Rs. 7, Rs. 5, Rs. 3 if he draws 3 balls of same colour, 2 balls of same colour, 1 ball of each colour respectively. Find his expectation.
D	A brochure inviting subscriptions for a new diet program states that the participants are expected to lose on an average 22 pounds in five weeks. Suppose that, from the data of the five-week weight losses of 26 participants, the sample mean and sample standard deviation are found to be 23.5 and 10.2, respectively. Could the statement in the brochure be substantiated based on these findings? Test at the α = 0.05 level of significance.
E	Evaluate using Green's theorem $\int_c (x^2ydx + y^3dy)$ where c is the boundary of the region bounded by $y = x^2$and $y = x$ from (0,0) to (1,1) then to (0,0) traversed in positive sense
F	Show that the vector, $\bar{F} = (x^2 - yz)i + (y^2 - xz)j + (z^2 - xy)k$ is irrotational and hence, find $\emptyset$ such that $\bar{F} = \nabla\emptyset$.
	5 marks each
Q3	**Solve any Four out of Six**
A	The IQs of individuals admitted to a state school for the mentally retarded are approximately normally distributed with a mean of 60 and a standard deviation of 10. (a) What is the probability that an individual picked at random will have an IQ between 55 and 75? (b) what is the lowest IQ of top 30% individuals?
B	If the mean age at death of 64 men engaged in an occupation is 52·4 years with standard deviation of 10·2 years, what are the 98% confidence limits for the mean age of all men in that population? Also determine can it be safely assume at 5% level of significance that that mean age of death of population is 56?
C	If the directional derivative of $\emptyset = ax^2$+by+2z at (1,1,1) is maximum in the direction of $i + j + k$, find a and b.
D	Evaluate $\int_c \frac{(12z-7)\,dz}{(z-1)^2(2z+3)}$, where c is the circle (i) $|z + i| = \sqrt{3}$

E

Use Stokes' theorem to evaluate $\int_c \bar{F} . \overline{dr}$ where $\bar{F} = (x^2 - y^2)i + 2xyj$ and c is the boundary of region bounded by $y = 0, x = 2, y = x$ in the xy plane.

F

For given the table of points

X	0	2	4	6	8	12	20
Y	10	12	18	22	20	30	30

Use normal equations, fit the straight line $y = ax + b$ to the data and find the value of $y(22)$.

Q4 **Solve any Four out of Six** **5 marks each**

A

In a study of the effectiveness of an insecticide against a certain insect, a large area of land was sprayed. Later the area was examined for live insects by randomly selecting squares and counting the number of live insects per square. Past experience has shown the average number of live insects per square after spraying to be 0.5. If the number of live insects per square follows a Poisson distribution, find the probability that a selected square will contain:

(a) One or more live insects

(b) Two live insects

B

On an average 20% of population in an area, suffer from T.B. What is the probability that out of 6 persons chosen at random from this area (a) at least 2, (b) none suffer from T.B.?

C

Evaluate $\int_c \bar{F} . \overline{dr}$ where $\bar{F} = yzi + (xz + 1)j + xyk$ along the line joining A (1,0,0) to B (2,1,4).

D

The following figures show the distribution of the digits in numbers chosen at random chosen from a telephone directory. Test at 5% level whether the digits may be taken to occur equally frequently in the directory.

Digits	0	1	2	3	4	5	6	7	8	9
Frequ.	1026	1107	997	966	1075	933	1107	972	964	853

E

Show that $\bar{F} = (y^2 - z^2 + 3yz - 2x)i + (3xz + 2xy)j + (3xy - 2xz + 2z)k$ is both irrotational and solenoidal.

F

Use divergence theorem to show that $\iint_s \bar{N} . \nabla r^2 \, ds = 6V$ where S is any enclosed surface enclosing volume V.