

S.E (CIVIL) / SEM-IV/R-19 / SA-2022 \ 08-12-2022

q.p. code:

10014587

(3 Hours)

[Total Marks: 80]

N.B.: 1) Question No. 1 is Compulsory

2) Answer any THREE questions from Q.2 to Q.6.

3) Figures to the right indicate full marks.

Q.1 (a) What is the value of $\int_0^{1+i} (x-y+ix^2) dz$ along the line from $z=0$ to $z=1+i$ (5)

(b) Find a and b such that $\vec{F} = (axy + z^3)i + x^2j + bz^2xk$ is irrotational (5)

(c) A random variable X has probability mass function $p(x) = kx^3$; $x=1,2,3,4$ then find the value of k, mean, variance. (5)

(d) Find the probability that at most 4 defective bulbs will be found in a box of 200 bulbs if it is known that 2% of the bulbs are defective. (5)

Q.2 (a) Find the rank correlation coefficient between X and Y (6)

X	17	13	15	16	6	11	14	9	7	12
Y	36	46	35	24	12	18	27	22	2	8

(b) A random variable has the MGF $M_X(t) = \frac{3}{3-t}$. Find mean and Variance of X. (6)

(c) Obtain Laurent's series expansions of $f(z) = \frac{z-1}{z^2-2z-3}$; $|z| > 3$. (8)

Q.3 (a) A coin is tossed. If it turns up heads two balls are drawn from urn A otherwise two balls are drawn from urn B. Urn A contains 3 black and 5 white balls. Urn B contains 7 black and one white ball. What is the probability that urn A was used, given that both balls drawn are black? (6)

(b) Fit a straight line $y = a + bx$ into the given data. (6)

x:	10	20	30	40	50
y:	22	23	27	28	30

(c) Prove that $\vec{F} = (6xy^2 - 2z^3)i + (6x^2y + 2yz)j + (y^2 - 6z^2x)k$ is irrotational. Find scalar potential of \vec{F} . Hence find the work done of moving particle from (1,0,2) to (0,1,1). (8)

- Q.4** (a) Using Green's Theorem evaluate $\int_c (xy + y^2)dx + x^2dy$ and c is closed curve of the region bounded by $y = x$ and $y = x^2$. (6)
- (b) A machinist is expected to make engine parts with axle diameter of 1.75 cm. A random sample of 10 parts shows a mean diameter of 1.85 cm, with a S.D of 0.1 cm. Based on this sample, would you say that the work of the machinist is inferior? (6)
- (c) A random variable X follows a normal distribution with mean 14 and standard deviation 2.5 find (1) $P[X < 8]$ (2) $P[X > 18]$ (3) $P[12 < X < 15]$ Given: Area between $z=0$ and $z=2.4$ is 0.4918 ; Area between $z=0$ and $z=1.6$ is 0.4452 ; Area between $z=0$ and $z=0.8$ is 0.2882 ; Area between $z=0$ and $z=0.4$ is 0.1554. (8)

- Q.5** (a) The standard deviation from two random samples of sizes 9 and 13 are 1.99 and 1.9. Can the samples be regard as drawn from normal population with same standard deviation? ($F_{(8,12)}(0.025) = 3.51, F_{(12,8)}(0.025) = 4.20$) (6)
- (b) Use Gauss's Divergence Theorem to evaluate $\iiint_S \vec{N} \cdot \vec{F} ds$, where $\vec{F} = 4xi - 2y^2j + z^2k$ and S is region bounded by $x^2 + y^2 = 4, z = 0, z = 4$. (6)
- (c) Obtain both Line of regressions for the data given below Given $\sum X = 250 ; \sum Y = 300 ; \sum XY = 7900 ; \sum X^2 = 6500 ; \sum Y^2 = 10000$ and $n = 10$ (in usual notation) (8)

- Q.6** (a) Evaluate Value of $\int_C \frac{\sin 2z dz}{(z + \pi/3)^4}$ where $C: |z| = 2$ (6)
- (b) The following data find the correlation coefficient to marks obtained by 11 students in 2 tests, one held at the beginning of the year and the other at the end of the year after intensive coaching: (6)

Test 1	19	23	16	24	17	18	20	18	21	19	20
Test 2	17	24	20	24	20	22	20	20	18	22	19

- (c) A die was thrown 132 times and the following frequencies were observed. (8)

No. obtained	1	2	3	4	5	6	Total
Frequency	15	20	25	15	29	28	132

Test the hypothesis that the die is unbiased at 5% level of significance.

(Given: Table value of χ^2 at 5% level of significance and 5 degree of freedom is 11.07)
