

1T00534 - S.E.(Chemical Engineering)(SEM-IV)(Choice Base Credit Grading System) (R- 19) (C Scheme) / 40321 - Engineering Mathematics-IV

QP CODE: 10040690

DATE: 08/12/2023

(3 Hours)

[Total Marks: 80]

N.B. : 1) Question No. 1 is Compulsory.

2) Answer any THREE questions from Q.2 to Q.6.

3) Figures to the right indicate full marks.

Q.1 (a) The mean life time of a sample of 25 bulbs is found as 1550 hours with standard deviation of 120 hours. The company of manufacturing bulbs claims that the average life of their bulbs is 1600 hours. Is the claim acceptable at 5% LOS ? (5)

(b) Find k and mean of following distribution (5)

X	8	12	16	20	24
P(X=x)	1/8	k	3/8	1/4	1/12

(c) Evaluate $\int_c z dz$ where c is unit circle $|z| = 1$ (5)

(d) Show that $\underline{F} = (y^2 - z^2 + 3yz - 2x)i + (3xz + 2xy)j + (3xy - 2xz + 2z)k$ is both solenoidal and irrotational. (5)

Q.2 (a) Evaluate $\int_c \frac{z+3}{(2z^2+3z-2)} dz$, where c is the circle $|z-i|=2$. (6)

(b) Fit a straight line for following data (6)

x	0	1	2	3	4
y	1	1.8	3.3	4.5	6.3

(c) Given $\underline{F} = (2xy + z)i + (x^2 + 2yz^3)j + (3y^2z^2 + x)k$, (8)

(a) Prove that \underline{F} is conservative.

(b) Find Scalar potential function ϕ such that $\underline{F} = \nabla\phi$.

(c) Find the work done by \underline{F} in moving a particle from $A(1, 2, 0)$ to $B(2,2,1)$ along the straight line AB.

Q.3 (a) Two different processes A and B are used to manufacture tubes. Samples were drawn from these two populations and following results were obtained (6)

Population	A	B
Sample Size	25	17
Sample SD	4	3

Test the hypothesis that variance of A lesser than variance of B.

Given $(F_{(24,16)}(0.05) = 2.24)$

(b) An I.Q. test was administered to 5 persons and after they were trained. The results are given below. Test whether there is any change in I.Q. after the training program, use 1% LOS. (6)

	I	II	III	IV	V
I.Q. Before training	110	120	123	132	125
I.Q. after training	120	118	125	136	121

(c) Find the Laurent's series for $f(z) = \frac{2z-3}{(z-1)(z-3)}$ about $z = 0$. (8)

- Q.4** (a) Using Green's Theorem evaluate $\int_c (xy + y^2)dx + x^2dy$ and c is closed curve of the region bounded by $y = x$ and $y = x^2$. (6)
- (b) Find the probability that at most 4 defective bulbs will be found in a box of 200 bulbs if it is known that 2% of the bulbs are defective. (6)
- (c) The following table gives the number of accidents in a district during a week. Apply χ^2 test to find whether the accidents are uniformly distributed over the week. (8)

Day	Sun	Mon	Tues	Wed	Thurs	Fri	Sat
No. of days	13	12	11	9	15	10	14

- Q.5** (a) Three factories A, B, C produce 30%, 50% and 20% of the total production of an item. Out of their production 80%, 50% and 10% are defective respectively. An item is chosen at random and found to be defective. Find the probability that it was produced by the factory A. (6)
- (b) A continuous random variable has pdf $f(x) = k(x - x^2), 0 \leq x \leq 1$. Determine k, mean, $P(0.5 \leq x \leq 3)$. (6)
- (c) The mean height of 500 students is 68 inches and the standard deviation is 4 inches. Assuming that the heights are normally distributed, find the number of students whose heights i) between 65 and 71 inches, ii) less than 62 inches, iii) greater than 72 inches. (8)

- Q.6** (a) Calculate Karl Pearson's coefficient of correlation from the following data. (6)

Price (in \$)	5	6	3	4	3
Demand (in units)	10	10	12	11	12

- (b) Use Divergence theorem to evaluate $\int \int_s \vec{F} \cdot \vec{ds}$ where $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ and s is the surface of the region $x^2 + y^2 = 4, z = 3$ above xy plane. (6)
- (c) The regression line of samples are $6y - 5x = 90$ & $15x - 8y = 130, \sigma_x = 4$ find i) Sample mean \bar{x}, \bar{y} ii) Coefficient of correlation between x & y iii) σ_y iv) Also estimate y at $x=10$. (8)
