Subject (Paper Code): 40321 / / Engineering Mathematics – IV

1T00534 / / S. E. (Chemical Engineering) (SEM – IV) (Choice Base Credit Grading System) (R - 19) (C Scheme)
1T00534 - S.E.(Chemical Engineering)(SEM-IV)(Choice Base Credit Grading System) (R- 20-21) (C Scheme) / 40321 - Engineering Mathematics-IV **Time: 3Hours** **Max. Marks: 80**

QP CODE:10012551   DATE: 08/12/2022

N. B.   **1. Question No. 1 is Compulsory**
2. Attempt any **Three** questions from remaining **Five** questions.
3. Figures to the right side indicate full marks.
4. Use of statistical tables is permitted.
5. Non – Programmable Calculator is allowed.

1. a) Find a, b, c if $\vec{F} = (axy + bz^3)i + (3x^2 - cz)j + (3xz^2 - y)k$ is irrotational.   **5**

b) Evaluate the following integral by Cauchy's residue theorem   **5**

$$\int_C \frac{e^{2z}}{(z-1)(z-2)} dz, whre\ c\ is\ the\ circle\ |z| = 3$$

c) A discrete random variable has the probability density function given below   **5**

x	-2	-1	0	1	2	3
y	0.2	k	0.1	2k	0.1	2k

Find k, Mean and Variance

d) Calculate Spearman's coefficient of rank Correlation form the following data.   **5**

X	12	17	22	27	32
Y	113	119	117	115	121

2. a) If $\bar{F} = 3x^2 i + 5xyj + xyz^3 k$ find $div\ \bar{F}$ and $curl\ \bar{F}$ at $(1,2,3)$ **6**

b) Evaluate $\int_0^{1+i}(x^2 + iy)dz$, along the path   **6**
  (i) $y = x$,
  (ii) $y = x^2$, is the line integral independent of path?

c) Calculate Karl Pearson's coefficient of Correlation for the following bivariate series.   **8**

X	28	45	40	38	35	33	40	32	36	33
Y	23	34	33	34	30	26	28	31	36	35

3. a) There are 11 tickets in a box bearing numbers 1 to 11. Three tickets are drawn one after the other without replacement. Find the probability that they are drawn in the order bearing (i) even, odd, even number, (ii) odd, odd, even number.   **6**

b) Fit a Poisson distribution to the following data   **6**

x	0	1	2	3	4
y	123	69	14	3	1

c) The 300 digits were chosen at random from a table of random numbers. The frequency of digits was as follows   **8**

Digit	0	1	2	3	4	5	6	7	8	9	Total
Frequency	28	29	33	31	26	35	32	30	31	25	300

Using $Ki\ square - test$ examine the hypothesis that the digits were distributed in equal numbers in the table

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4. **a)** Find the total work done in moving a particle in the force field $\bar{F} = 3xyi - 5zj + 10xk$ along $x = t^2 + 1, y = 2t^2, z = t^3$ from $t = 1\ to\ t = 2$ **6**

b) Evaluate $\oint_c \frac{\sin\pi z^2 + \cos\pi z^2}{(z-2)(z-1)}\, dz$ where C is the circle $|z| = 4$.   **6**

c) Find the lines of regression for the following data   **8**

x	70	72	74	76	78	80
y	163	170	179	188	196	220

5. **a)** Find the normalizing factor, k if the following function is a probability density function $f(x) = k(1 - x^2), 0 < x < 1$ .Also find $P(0.1 < x < 0.2) and\ P(x > 0.5)$ **6**

b) For a normal variate with mean 2.5 and standard deviation 3.5, find the probability that $(i) 2 \leq x \leq 4.5,\ (ii) -1.5 \leq x \leq 5.5$ **6**

c) The number of car accidents in a metropolitan city was found to be 20, 17, 6, 7, 15, 8, 5, 16 and 14 per month respectively. Use $Ki\ square - test$ to cheek whether these frequencies are in agreement with the belief that occurrence of accidents was the same for 10 months period. Test at 5% level of significance.   **8**

6. **a)** Evaluate by Green's Theorem $\int \bar{F}.dr$ where $\bar{F} = x^2i - xyj$ over the region of triangle having vertices $A = (0,2), B(2,0)\ and\ C(4,2)$ **6**

b) Nine items of a sample had the following values 45, 47, 50, 52, 48, 47, 49, 53, 51 Does the mean of 9 items differ significantly from the assumed population mean 47.5 ?   **6**

c) Two batches of 12 animals each are given test of inoculation, One batch was inoculated and the other was not. The number of dead and surviving are given in the following table for both cases. Can the inoculation be regarded as effective against the disease at 5% level of significance (make Yates -Correction)   **8**

	Dead	Surviving	Total
Inoculated	2	10	12
Not - Inoculated	8	4	12
Total	10	14	24

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