1T00533 - S.E.(Chemical Engineering)(SEM-III)(Choice Base Credit Grading System) (R- 19) (C Scheme) / 50721 - Engineering Mathematics - III

QP CODE:10011432 **[Time: 3 Hours]** DATE: 21/11/2022. **[Marks:80]**

Please check whether you have got the right question paper.

N.B:
1. Question.No.1 is compulsory.
2. Solve any three questions from the remaining questions.
3. Non programmable calculator is allowed.

Q.1 a) Find Laplace transform of $f(t) = e^{4t}(sin^3 3t + cosh^3 3t)$ **(05)**

b) Find half range cosine series of $f(x) = sinx$ in the interval $(0, \pi)$. **(05)**

c) Show that the Eigen values of $A = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$ is of unit modulus. **(05)**

d) Test whether the function $f(x) = ize^{-iz}$ is analytic? **(05)**

Q.2 a) Show that the function $u(x, y) = 3x^2y + 2x^2 - y^3 - 2y^2$ is harmonic and hence find its conjugate harmonic function. **(06)**

b) Solve $\frac{\partial^2 u}{\partial x^2} - 16\frac{\partial u}{\partial t} = 0$ subject to the condition $u(0,t) = 0, u(1,t) = 3t, u(x,0) = 0, 0 \le x \le 1,$. Taking h = 0.25 upto 3 seconds only by using. Bender-Schmidt method. **(06)**

c) Find a) $L^{-1}\left\{\frac{5s^2-7s+17}{(s-1)(s^2+4)}\right\}$ b) $L^{-1}\left\{tan^{-1}\left(\frac{2}{s}\right)\right\}$ **(08)**

Q.3 a) If $A = \begin{bmatrix} y & y \\ y & y \end{bmatrix}$, prove that $e^A = e^y\begin{bmatrix} coshy & sinhy \\ sinhy & coshy \end{bmatrix}$ **(06)**

b) Find the Fourier series of $f(x) = 2x - x^2$ in the interval (0, 3), hence deduce that $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots\ldots$ **(06)**

c) Find the solution of one dimensional heat flow is given by $\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2}$ for which $u(0,t) = 0, u(l,t) = 0, u(x,0) = 100\frac{x}{l}$. **(08)**

Q.4 a) Using convolution theorem, find the inverse Laplace transform of $f(s) = \frac{S+29}{(S+4)(S^2+9)}$ **(06)**

b) Test whether the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ is similar to a diagonal matrix, if yes, write the diagonal matrix. **(06)**

c) Find the Fourier series of $f(x) = 1 + \frac{2x}{\pi}, -\pi < x < 0$ **(08)**

$= 1 - \frac{2x}{\pi}, 0 < x < \pi$

and hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots\ldots$

Q.5 a) Use the separation of variables techniques to solve $3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0$ with $u(x,0) = 4e^{-x}$ **(06)**

b) Evaluate $\int_0^\infty e^{-3t}\, t^2 \sinh 2t\, dt$. **(06)**

c) Verify $C - H$ theorem for $A = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$ and evaluate $2A^4 - 5A^3 - 7A + 6I$. **(08)**

Q.6 a) Find the Fourier series of $f(x) = \left(\frac{\pi - x}{2}\right)^2$ in $0 \le x \le 2\pi$. **(06)**

b) Find the orthogonal trajectories of the family of curves $x^4 - 6x^2y^2 + y^4 = c_1$ **(06)**

c) If $u - v = \frac{\cos x + \sin x - e^{-y}}{2(\cos x - \cosh y)}$ and $f\left(\pi/2\right) = 0$ then determine the analytic function $f(z) = u + iv$ **(08)**

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