(3 Hours) **[Total Marks 100]**

N. B.:
1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Draw neat diagrams wherever necessary.
4. Symbols have usual meaning unless otherwise stated.
5. Use of non-programmable calculator is allowed.

Q 1. Attempt any **two**:

(i) State and prove Kepler's laws of planetary motion. 10

(ii) Obtain the equation of motion of a particle of mass 'm' as related to the rotating earth. Hence show that the angle between g and g_e depends on colatitude angle ϴ. 10

(iii) Derive an expression for Coriolis theorem. Interpret each term. 10

Q 2. Attempt any **two**:

(i) What is meant by generalized co-ordinates? Derive an expression for generalized velocity and generalized kinetic energy. 10

(ii) Obtain Lagrange's equation of motion by using D'Alembert's principle for holonomic systems. 10

(iii) A double pendulum consists of two weightless rods connected to each other and a point of support. The masses m_1 and m_2 are not equal but the length of the rods are equal. Pendulums are free to swing only in one vertical plane. Derive the Lagrangian for the system. 10

Q 3 Attempt any **two**:

(i) For fluids, derive the conservation equation for energy in the form of Bernoulli's theorem. 10

(ii) Derive Euler's equations of motion of a rigid body. Solve these equations for a torque free rotational motion of symmetric body and hence show that the magnitude of the angular velocity vector is a constant. 10

(iii) A rigid body is constrained to move about a point which is fixed. Derive an expression for its angular momentum about the instantaneous axis of rotation passing through the fixed point. Hence derive an expression for kinetic energy, $T = \frac{1}{2}\vec{\omega}.\vec{L}$. 10

Q 4 Attempt any **two**:

(i) The potential energy of a one dimensional anharmonic oscillator is given by $V(x) = K\left(\frac{x^2}{2} + \frac{\alpha x^4}{4}\right)$, where K is the spring constant and α is anharmonic coefficient. Discuss the potential energy curve for various combinations of K and α. 10

(ii) Discuss fixed points of a logistic map, stability of fixed points and periodic attractors. Explain how series of bifurcations lead to chaos when $\lambda = 3.57$. 10

(iii) Discuss numerical solutions of Duffing's equation for $\gamma = 0.1$, and $f = 0.5$ & $f = 3$ and also explain why the two solutions for even harmonics are different. 10

Q 5. Attempt any **four**:

(i) Show that when a body moves in a central force field its motion is confined to a plane. 5

(ii) Find the Coriolis force acting on a body of mass 1kg moving northward with a horizontal velocity of 100 m/s at a place on earth whose colatitudes is 60° N 5

(iii) Define constraints. With good examples, explain holonomic and non-holonomic constraints. 5

(iv) Define cyclic coordinate. Which coordinate is called as cyclic coordinate in equation given below 5

a) $L = \frac{1}{2}\, m\, (\dot{x}^2 + \dot{y}^2) - mgy$

b) $p_\theta = mr^2\dot{\theta}$

(v) Consider a fluid flow in which velocity is given by, $\vec{v}(x,t) = \frac{at}{x}\hat{\imath},\ x > 0$. 5
Find the acceleration $\vec{a}(x,t)$ of the fluid element at position x and time t.

(vi) If a rigid body consists of three particles of masses 2, 1 and 4 grams located at (+1 , -1 , 1) , (2 , 0 , 2) (-1 , 1 , 0) cm respectively. Find principal moment of inertia and product of moment of inertia. 5

(vii) Two very close initial values of x on logistic map are 0.50000 and 0.50002 respectively. With λ = 4 after 20 iterations the values are 0.08561 and 0.00561 respectively. Calculate Lyapunov exponent. 5

(viii) What is phase space diagram? Plot the phase space diagram for one dimensional oscillator. 5
