

Duration:[3 Hours]

[Total Marks: 100]

- N.B. 1) All questions are compulsory.
2) Figures to the right indicate full marks.

1. Choose correct alternative in each of the following:

(20)

- i. Which of the following statement is true?
 - (a) In a simple graph there are at most two vertices of equal degree
 - (b) In a simple graph there at least two vertices of equal degree
 - (c) Degree of vertices are all distinct
 - (d) None of these
- ii. The number of edges of Complete Bipartite graph $K_{m,n}$ is
 - (a) $m + n$
 - (b) mn
 - (c) $m^2 + n^2$
 - (d) None of these
- iii. Adjacency matrix of a simple graph G is
 - (a) Binary and symmetric
 - (b) Only binary
 - (c) Only Symmetric
 - (d) None of these
- iv. Let G be a graph on 8 vertices that has no pair of adjacent edges. The degree sequence of G is
 - (a) 2,2,2,2,1,1,1,1;
 - (b) 1,1,1,1,1,1,1,1;
 - (c) 2,2,1,1,1,1,1,1;
 - (d) such a graph does not exist.
- v. A $G(p, p - 1)$ graph is a tree if
 - (a) G is connected
 - (b) G is acyclic
 - (c) Must be (a) or (b)
 - (d) None of the above
- vi. Let T be a tree with n vertices, where $n \geq 4$ and let v be a vertex of maximum degree in T , then T is a path if $d(v)$ is
 - (a) 3
 - (b) 2
 - (c) 1
 - (d) $n - 1$
- vii. If v is a cut vertex of a graph G , then
 - (a) G is Hamiltonian and Eulerian.
 - (b) G is Hamiltonian but not Eulerian
 - (c) G is not Hamiltonian.
 - (d) None of the above.
- viii. Complete bipartite graph is Eulerian if
 - (a) $|V_1| = |V_2|$
 - (b) $|V_1| > |V_2|$
 - (c) $|V_1| = |V_2| = \text{Even Number}$
 - (d) None of these
- ix. If G is Hamiltonian and if S is any non empty proper subset of $V(G)$ then
 - (a) $\omega(G - S) = |S|$
 - (b) $\omega(G - S) \leq |S|$
 - (c) $\omega(G - S) \geq |S|$
 - (d) None of these
- x. Number of edges in cube graph Q_4 is
 - (a) 24
 - (b) 16
 - (c) 18
 - (d) 32

[TURN OVER

2. (a) Attempt any **ONE** question from the following: (8)
- i. State and prove *Havel – Hakimi* theorem for degree sequence of a graph G .
 - ii. If $(A^n) = (a_{ij}^n)$ is the n^{th} power of adjacency matrix A of a graph G with $V(G) = \{v_1, v_2, \dots, v_n\}$, then prove that
 - (a) $a_{ij}^2, i \neq j$ is the number of $v_i - v_j$ path of length 2.
 - (b) $a_{ii}^2 = \text{deg}(v_i)$
 - (c) $\frac{1}{6}\text{trace of } A^3$ is the number of triangles in G .

- (b) Attempt any **TWO** questions from the following: (12)
- i. State and prove Hand-Shaking theorem for Graph. Hence prove that every graph has an even number of odd vertices
 - ii. For any graph G with at least 6 vertices, prove that either G or G^c contains a triangle.
 - iii. If G is simple graph with p vertices, q edges and k components, then prove that $q \geq p - k$.
 - iv. If G is graph of order p with $\delta(G) \geq (p - 1)/2$, then show that G is connected. Is the bound $(p - 1)/2$ sharp?, that is, in this case, can $(p - 1)/2$ be replaced by $(p - 2)/2$?

3. (a) Attempt any **ONE** question from the following: (8)
- i. Let G be a (p, q) graph. Prove that following statements are equivalent.
 - a) G is tree.
 - b) G is acyclic and $q = p - 1$.
 - c) G is connected and $q = p - 1$.
 - ii. State and prove Cayley's formula for spanning trees.

- (b) Attempt any **TWO** questions from the following: (12)
- i. Show that there exist a tree with degree sequence $d_1 \geq d_2 \geq \dots \geq d_n$ if and only if a) $d_i \geq 1$, for $1 \leq i \leq n$ and b) $\sum_{i=1}^n d_i = 2n - 2$
 - ii. Define a cut vertex of a graph G . Show that vertex v is a cut vertex if and only if there exists two vertices x and y such that v is on every $x - y$ path in G .
 - iii. Explain and write Huffman's algorithm for prefix code.
 - iv. Write Breadth First Search and Depth First Search Algorithm for finding a spanning tree.

4. (a) Attempt any **ONE** question from the following: (8)
- i. Show that a nontrivial connected graph G is Eulerian if and only if every vertex of G has even degree.
 - ii. If G is graph on p vertices with $p \geq 3$ and $\delta(G) \geq \frac{p}{2}$ where $\delta(G)$ denotes the minimum degree of G then show that G contains a Hamiltonian cycle.

[TURN OVER

(b) Attempt any **TWO** questions from the following: (12)

- i. Define a line graph $L(G)$ of a graph G . Show that if the simple graphs G_1 and G_2 are isomorphic, then its line graphs $L(G_1)$ and $L(G_2)$ are isomorphic. Is converse true? Justify.
- ii. Let G be a simple graph with $p \geq 3$. If closure of G is complete, show that G is Hamiltonian.
- iii. Prove that the cube graph Q_k is bipartite k -regular graph with 2^k vertices.
- iv. If G is a (p, q) graph with $p \geq 3$ and $q \geq \frac{1}{2}(p-1)(p-2) + 2$, then prove that G is Hamiltonian.

5. Attempt any **FOUR** questions from the following: (20)

- (a) Show that a graph G is disconnected if and only if its vertex set V can be partitioned into two subsets V_1 and V_2 such that there exists no edge in G whose one end vertex is in the subset V_1 and the other in the subset V_2 .
- (b) Show that every $u - v$ walk W contains $u - v$ path.
- (c) Prove that a connected graph G is a tree if and only if every edge of G is a cut edge.
- (d) Show that every two vertices of a tree are connected by a unique path.
- (e) If G is Hamiltonian graph then for every nonempty proper subset S of $V(G)$, prove that $\omega(G - S) \leq |S|$. Is converse true? Justify.
- (f) Describe Fluery's Algorithm to find a closed Eulerian trail.
