

(3 hours)

[Total Marks : 80

N. B.

1. Question no. 1 is compulsory.
2. Answer any three out of the remaining five questions.
3. Assumption made should be clearly stated.
4. Assume any suitable data wherever required but justify the same.
5. Illustrate the answers with sketches wherever required.
6. **Answer to the questions should be grouped and written together.**

Q1 Write the short notes on(ANY FIVE):

20

- a. Pre-Processing, Processing and Post-Processing in FEA
- b. Sub parametric, Iso-parametric and super parametric element in FEA.
- c. Geometric and Forced boundary condition
- d. Advantages and limitations of the FEM.
- e. Write element matrix equation in the following fields. Explain each term properly.
 - (i) 1D steady state, heat transfer by conduction
 - (ii) 1D, steady state steady flow of fluid in a pipe
- f. Sources of Error in FEA.

Q2 a. Solve the following differential equation using Galerkin Method.

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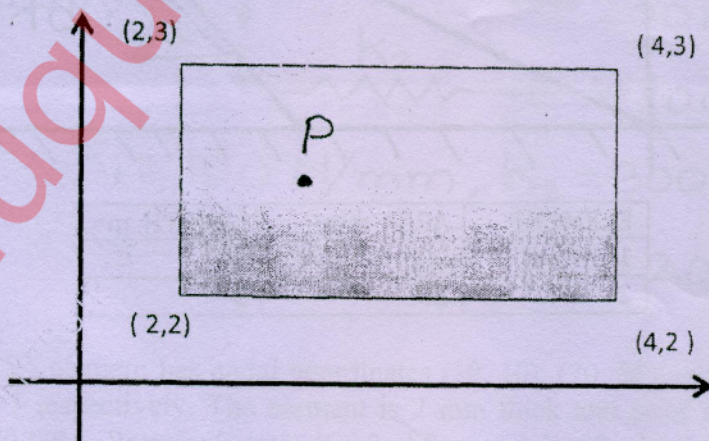
$$-\frac{d}{dx} \left[(x-1) \frac{du}{dx} \right] = x^2 ; 3 \leq x \leq 5$$

Boundary conditions: $u(5) = 10$ and $u'(3) = 5$

Compare the answers with exact solution at $x = 4$ and 5 .

b. Compute the temperature at point P (2.5, 2.5) using natural coordinates system for quadrilateral element shown in the figure.

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Take: $T_1 = 100^\circ \text{C}$

$T_2 = 60^\circ \text{C}$

$T_3 = 50^\circ \text{C}$

$T_4 = 90^\circ \text{C}$

- Q3 a. A copper fin of diameter 20mm, length 60mm and thermal conductivity is 15
 100 W/m⁰ C and is exposed to ambient air at 30⁰ C with a heat transfer coefficient
 25 W/m² 0 C. If one end of the fin is maintained at temperature 500⁰C and other
 end is at 200⁰ C. Solve the following differential equation for obtaining the
 temperature distribution over the length of a fin.

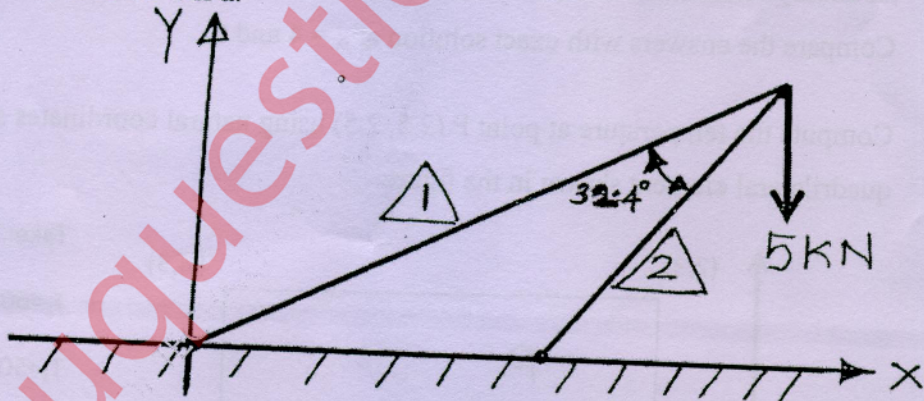
$$kA \cdot \frac{d^2\theta}{dx^2} - hp\theta = 0$$

$\theta =$ Temperature difference = $T_x - T_a$.

Use Rayleigh-Ritz method, mapped over general element, taking Lagrange's linear
 shape functions and two linear elements.

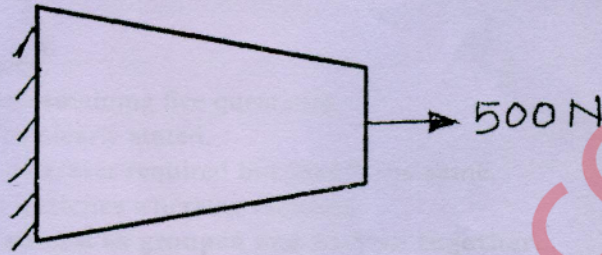
Write all the steps clearly. Compare your answer with exact at $x = 20, 40$ mm

- b. What do you mean by consistent and lumped mass matrices? Derive the same for 5
 linear bar element. 10
- Q4 a. Find the natural frequency of axial vibration of a bar having cross sectional area as
 30 x 10⁻⁴ m², 1 m length with left end fixed. Take $E = 2 \times 10^{11}$ N/m². Density of the
 material is 7800 Kg/m³. Take two linear elements. 10
- b. Analyze the truss completely for displacement and stress as shown in figure. 10
 Take: $E = 2 \times 10^5$ MPa.

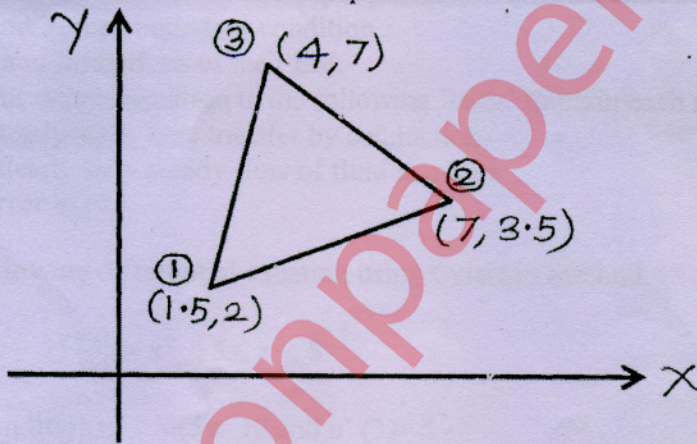


ELEMENT	AREA, mm ²	LENGTH, m
1	20	6
2	20	3

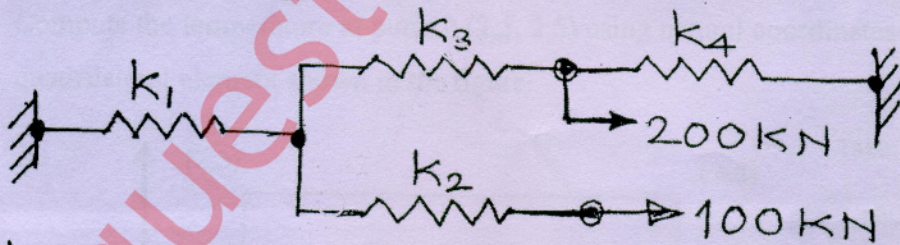
- Q5 a. Using FEM, analyze the taper bar as shown in figure. The cross sectional area to the left and right is equal to 80 mm^2 and 20 mm^2 . Take length of bar is equal to 60 mm , Take $E = 210 \text{ GPa}$. 10



- b. Evaluate the shape function and prove its property, for triangular element as shown in figure. Also sketch the variation of shape function for each node. 10



- Q6 a. Determine the displacement at nodes by using the principal of minimum potential energy and find the support reaction. 10



Use, $k_1 = 100 \text{ N/mm}$, $k_2 = 300 \text{ N/mm}$,
 $k_3 = 150 \text{ N/mm}$, $k_4 = 200 \text{ N/mm}$.

- b. A CST element has nodal coordinates $(10, 10)$, $(70, 35)$ and $(75, 25)$ for nodes 1, 2 and 3 respectively. The element is 2 mm thick and is of material with properties $E = 70 \text{ GPa}$. Poisson's ratio is 0.3 . After applying the load to the element the nodal deformation were found to be $u_1 = 0.01 \text{ mm}$, $v_1 = -0.04 \text{ mm}$, $u_2 = 0.03 \text{ mm}$, $v_2 = 0.02 \text{ mm}$, $u_3 = -0.02 \text{ mm}$, $v_3 = -0.04 \text{ mm}$. Determine the strains ϵ_x , ϵ_y , ϵ_{xy} and corresponding element stresses. 10