(3 Hours) **Max Marks: 80**

Note:
1. **Question No. 1 is compulsory.**
2. **Out of remaining questions, attempt any three questions.**
3. **Assume suitable additional data if required.**
4. **Figures in brackets on the right hand side indicate full marks.**

Q.1. (A) State Central limit theorem and give its significance. (05)

(B) State the three axioms of probability. (05)

(C) State various properties of autocorrelation function and power spectral density function. (05)

(D) State and explain Bayes Theorem. (05)

Q.2. (A) A random variable has the following exponential probability density function: $f(x) = Ke^{-|x|}$. Determine the value of K and the corresponding distribution function. (10)

(B) A distribution has unknown mean μ and variance 1.5. Using Central Limit Theorem find the size of the sample such that the probability that difference between sample mean and the population mean will be less than 0.5 is 0.95. (10)

Q.3. (A) Explain Ergodicity in Random Process. (10)
A Random process is given by $X(t) = 10cos(50t + Y)$
where ω is constant and Y is a Random variable that is Uniformly distributed in the interval $(0, 2\pi)$. Show that $X(t)$ is a WSS process and it is Correlation ergodic.

(B) If X and Y are independent Random variables and if $Z=X+Y$, then show that the pdf of Z is given by the convolution of the pdf of X and pdf of Y. (10)

Q.4. (A) The transition probability matrix of Markov Chain is given by, (10)

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.5 & 0.4 & 0.1 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{bmatrix} \end{array}$$

Find the limiting probabilities?

(B) Explain Strong law of large numbers and weak law of large numbers. (05)

(C) If A and B are two independent events then prove that $P(A \cap \bar{B}) = P(A).P(\bar{B})$. (05)

Q.5. (A) State and prove Chapman-Kolmogorov equation. (10)

(B) In a communication system a zero is transmitted with probability 0.4 and a one is transmitted with probability 0.6. Due to noise in the channel a zero can be received as one with probability 0.1 and as a zero with probability 0.9, similarly one can be received as zero with probability 0.1 and as a one with probability 0.9. If one is observed, what is the probability that a zero was transmitted? (10)

Q.6. (A) Explain power spectral density function. State its important properties and prove any two of the property. (10)

(B) Explain (i) M/G/1 Queuing system. (05)
(ii) M/M/1/∞ Queuing system.

(C) Write short notes on Gaussian distribution. (05)

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