**Q. P. Code: 32505**

**(3 Hours)** **Max Marks: 80**



**Note:**
1. **Question No. 1 is compulsory.**
2. **Out of remaining questions, attempt any three questions.**
3. **Assume suitable additional data if required.**
4. **Figures in brackets on the right hand side indicate full marks.**

1. (A) State Central limit theorem and give its significance (05)
   (B) State the three axioms of probability. (05)
   (C) State and explain Bayes Theorem. (05)
   (D) Define Power spectral density and prove any two properties. (05)
2. (A) Prove that if input to LTI system is w.s.s. then the output is also w.s.s. (10)
   (B) In a factory, four machines $A_1$, $A_2$, $A_3$ and $A_4$ produce 35%, 10%, 25% and 30% of the items respectively. The percentage of defective items produced by them is 3%, 5%, 4% and 2%, respectively. An item is selected at random. (10)
   (i) What is the probability that the selected item will be defective?
   (ii) Given that the item is defective what is the probability that it was produced by machine $A_4$?
3. The joint probability density function of two random variables is given by (20)
   $f_{x,y}(x,y) = 15e^{-3x-3y} : x \geq 0, y \geq 0$
   i) Find the probability that $x<2$ and $y>0.2$.
   ii) Find the marginal densities of $x$ and $y$.
   iii) Are $x$ and $y$ independent?
   iv) Find $E(x/y)$ and $E(y/x)$.
4. (A) A stationary process is given by $X(t) = 10\cos[100t + \theta]$ where $\theta$ is a random variable with uniform probability distribution in the interval $[-\prod, \prod]$. Show that it is a wide sense stationary process. (10)
   (B) Explain Strong and weak law of large numbers. (05)
   (C) Write short notes on the following special distributions. (05)
   i) Uniform distribution.
   ii) Gaussian distribution.
5. (A) Define discrete and continuous random variables by giving examples. Discuss the properties of distribution function. (10)
   (B) A random variable has the following exponential probability density function: $f(x) = Ke^{-|x|}$. Determine the value of $K$ and the corresponding distribution function. (10)
6. (A) Suppose $X$ and $Y$ are two random variables. Define covariance and correlation of $X$ and $Y$. When do we say that $X$ and $Y$ are (10)
   (i) Orthogonal,
   (ii) Independent, and
   (iii) Uncorrelated?
   Are uncorrelated variables independent?
   (B) State and prove Chapman-Kolmogorov equation. (10)

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