V / RSA / EXTC / CBGS / 22.11.16

Q.P. Code : 587802

Random Signal Analysis

(03 Hrs.) **Total Marks: 80**

N.B.:

(1) **Question No. 1 is Compulsory**
(2) **Attempt any Three questions from the remaining Five questions**
(3) Assume suitable data if necessary

Q1. (a) Explain any two properties of autocorrelation function. 05

(b) State and explain Chebyshev's inequality. 05

(c) State Central Limit theorem and give its significance. 05

(d) State and explain Bayes' theorem. 05

Q2. (a) A two dimensional Random variable has the following pdf, 10

$$f_{XY}(x,y) = kxye^{-(x^2+y^2)},\ x \geqslant 0, y \geqslant 0$$

Find
(i) Value of constant K.
(ii) Marginal density of X and Y.
(iii) Conditional densities of X and Y.
(iv) Check for independence of X and Y.

(b) In a communication system, a zero is transmitted with probability 0.3 and a one is transmitted with probability 0.7. Due to noise in the channel, a zero is received as one with probability 0.2 . Similarly, a one is received as zero with probability 0.4. Now, 10
(i) What is the probability that a one is received?
(ii) It is observed that a one is received. What is the probability that zero was transmitted?
(iii) What is the probability that an error is committed?

Q3. (a) If the joint pdf of (X,Y) is given as , 10

$$f_{XY}(x,y) = e^{-(x+y)}\quad x \geqslant 0, y \geqslant 0$$

Find the probability density function of (U,V), where $U = \frac{X}{X+Y}$ and V= X+Y.
Are U and V independent?

(b) Define Moment Generating function of a Random variable. If X is a RV discrete or Continuous, then show that its nth raw moment is given as, 10

$E(X^n) = \frac{d^n Mx(t)}{dt^n}$ at t=0.

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Q4. (a) Let X1,X2,X3,..... be sequence of Random variables. 10

Define (i) Convergence almost everywhere

(ii) Convergence in probability

(iii) Convergence in distribution

(iv) Convergence in mean square sense

for the above sequence of Random variable X.

(b) Prove that if input to an LTI system is WSS process , then its output is also a WSS process. 10

Q5. (a) A Random process is given by X(t)= A cos (ωt + θ), where A and ω are constants and θ is a Random variable that is Uniformly distributed in the interval (0, 2π). Show that X(t) is a WSS process and it is Correlation ergodic. 10

(b) Explain Power spectral density and prove any two of its properties. 10

The power spectrum of a WSS process is given by,

$$S(\omega) = \frac{10\omega^2+25}{(\omega^2+4)(\omega^2+9)}$$

Find its autocorrelation function

Q6. (a) State and prove Chapman-Kolmogorov equation 10

(b) The transition probability matrix of a Markov chain {Xn} n=1,2,...., having three states 1,2 and 3 is, 10

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix} \end{array}$$

The initial probability distribution is $p^{(0)} = (0.7,\ 0.2\ ,0.1)$

Find (i) $P(X_2 = 3$

(ii) $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$

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