(REVISED COURSE)

(3 Hours) **[Total Marks: 80**

N.B. :

1) **Question 1 is compulsory**. Answer any **three** more from the remaining questions.
2) Assume data if necessary and **specify the assumptions** clearly.
3) Draw neat sketches wherever required.
4) Answers to the sub-questions of an individual question should be grouped and written together i.e. one below the other.

1. (a) Analyze the following LP: **[05]**

$$\begin{aligned} Max \;\; z &= x_1 - 2x_2 + x_3 \\ s.t.\; 2x_1 - x_2 + 3x_3 &\leq 4 \\ x_1 - 0.5x_2 + x_3 &\leq 6 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

(b) A primal can be written in matrix form, with usual notation, as: **[05]**

$$\begin{aligned} Max \; z &= c^T x \\ s.t.\; Ax &\leq b \\ x &\geq 0 \end{aligned}$$

Show that $w \geq z$, where w is the objective function of the dual.

(c) The following table gives the time required in minutes, by three mechanics, on three different machines, for an assembly job: **[05]**

Mechanic	*Machines*		
	Lathe	*Milling*	*Welding*
M1	t_{11}	t_{12}	t_{13}
M2	t_{21}	t_{22}	t_{23}
M3	t_{31}	t_{32}	t_{33}

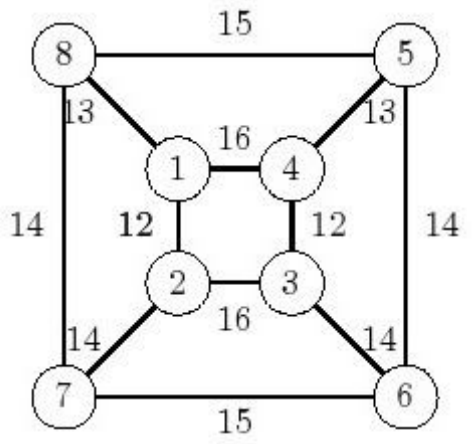
Formulate this assignment problem as an LP to decide how the duties must be allocated so as to minimize the total time.

(d) The time between arrivals at the State Revenue Office is exponential with a mean value of 0.04 hr. The office opens at 8 : 00 $A.M$. Find the probability that no customers will arrive at the office by 8 : 30 $A.M$. **[05]**

2. (a) Solve the following LP using the Two-Phase Simplex: **[10]**

$$\begin{aligned} Min \;\; z &= x_1 + x_2 \\ s.t.\; 2x_1 + 4x_2 &\geq 8 \\ 4x_1 + 2x_2 &\geq 8 \\ x_1, x_2 &\geq 0 \end{aligned}$$

(b) Use Prim's algorithm to find the minimum spanning tree of the following graph: **[10]**



3. (a) Solve the following IP using Branch and Bound Technique: **[10]**

$$Max \quad z = 1.5x_1 + 2x_2$$
$$s.t. \; 2x_1 + 3x_2 \leq 14$$
$$2.5x_1 + 2x_2 \leq 12$$
$$x_1, x_2 \geq 0 \;\; and \; integer$$

(b) An investor has 4000\$ at her disposal for investing in three mutual funds, where investments can be made in lots of 1000\$ only. Returns from each policy for various levels of investment are given below: **[10]**

Policy	*Investmrnt in* \$1000			
	1	2	3	4
P1	1.20	2.05	3.75	4.80
P2	1.10	2.40	3.50	4.75
P3	1.15	2.50	3.35	4.85

The figures in the table are in thousands of dollars. Use Dynamic Programming to determine the optimum investment policy to maximize the total returns.

4. Solve the following transportation problem using Vogel's Approximation to obtain the initial BFS: **[20]**

Source	*Dest. #1*	*Dest. #2*	*Dest. #3*	*Cap.*
1	20	18	16	225
2	18	20	22	810
3	21	24	24	215
Demand	360	420	560	

The entries represent cost of transportation in rupeees from each supply node to each destination node.

5. A company produces two models of electronic gadgets that use resistors, capacitors, and chips. The following table summarizes the data: **[20]**

	Requirements per unit		
Resource	*Model A*	*Model B*	*Max. Available*
Resistors	2	3	1200
Capacitors	2	1	1000
Chips	0	4	800

The unit profit for *Model A* is \$3, and that for *Model B* is \$4.

(a) Determine the range of applicability of the dual prices for each resource.

(b) If the available number of resistors is incresed to 1300 units, find the new optimum.

(c) If the avilability of the capacitors is limited by the range of applicability computed above in the first part, determine the corresponding range of the optimal profit and corresponding ranges for the number of units to be produced of *Model A and Model B*.

(d) A new contractor is offering to sell resistors at 40 *cents* eash, but only if the company would purchase at least 500 units. Should the company accept the offer?

6. (a) Solve the following LP using the Revised Simplex Method: **[10]**

$$\begin{aligned} Max \;\; z &= 2x_1 + x_2 + 4x_3 \\ s.t. \;\; x_1 + x_2 + x_3 &\leq 10 \\ x_1 - x_2 + 2x_3 &\leq 16 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

(b) For the upcoming planting season, a farmer has three options to choose from: he can plant corn (option A_1), or he can plant wheat (option A_2) , or he can also plant soybeans (option A_3), or even leave the land for grazing(option A_4). The payoffs associated with the different options are influenced by the amount of rain: heavy rainfall(S_1), moderate rainfall(S_2), light rainfall(S_3), or drought season(S_4). The payoff matrix, in thousands of dollars, is estimated as: **[10]**

Option\State	S_1	S_2	S_3	S_4
A_1	-20	60	30	-5
A_2	40	50	35	0
A_3	-50	100	45	-10
A_4	12	15	15	10

Using Minimax strategy determine the optimum policy for the farmer.

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