

(REVISED COURSE)
 (3 Hours)

[Total Marks: 80

N.B. :

- 1) **Question 1 is compulsory.** Answer any **three** more from the remaining questions.
- 2) Assume data if necessary and **specify the assumptions** clearly.
- 3) Draw neat sketches wherever required.
- 4) Answers to the sub-questions of an individual question should be grouped and written together i.e. one below the other.

1. (a) The flow of a particular fluid is described by its velocity components as: [05]

$$u = -\frac{V_0}{l}x, \quad v = -\frac{V_0}{l}y$$

where V_0 and l are constants. Derive an expression to calculate the rate of change of the density of the fluid $\rho(x, y, z, t)$ with respect to time, along a flowing fluid particle.

- (b) Consider the following equation: [05]

$$\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} = 0$$

Derive an expression to solve the equation numerically using an implicit scheme.

- (c) Consider the following function: [05]

$$f(x, y) = \exp(-x^2) + \exp(-y^2)$$

Use backward differences to calculate:

$$\frac{\partial f(x, y)}{\partial x} \text{ and } \frac{\partial f(x, y)}{\partial y}$$

at $x = 0.1$, and $y = 0.1$. Also calculate the error. Take

$$\Delta x = 0.05 \text{ and } \Delta y = 0.05$$

- (d) Write a short note on the upwind scheme. [05]

2. (a) An insulated metal rod has an initial temperature profile given by: [10]

$$T(x, 0) = 20^\circ C$$

At time $t = 0$, a hot reservoir at a temperature of $T = 200^\circ C$ is brought into contact with the left end of the rod. Simultaneously the right end is exposed to a hot reservoir at a temperature of $300^\circ C$. The length of the rod is 1 m , and for the metal $\alpha = 1.0 \times 10^{-5} \text{ m}^2/\text{sec}$. Using the finite difference FTCS scheme, calculate the temperature across the rod for the next 1600 sec . Assume $r = 0.2$ and $\Delta x = 0.2 \text{ m}$.

- (b) Solve the following equation using the weighted residual method: [10]

$$\frac{d^2 u}{dx^2} = -(x + 1)$$

with $u = 0$ at $x = 0$ and $\frac{du}{dx} = 1$ at $x = 2$ Use the trial function $u = a_1 x + a_2 x^2$

3. Consider the steady, one dimensional heat conduction, in an insulated metal rod, of length 0.8 m . The ends of the rod are maintained at constant temperatures of 200°C and 400°C respectively. Applying the finite volume method on eight control volumes of equal length, determine the temperature profile across the rod. Thermal conductivity of the metal is $k = 1000 \text{ W/m.K}$ and the cross-sectional area of the rod is $A = 0.01 \text{ m}^2$ [20]
4. (a) A company produces a perishable product in a factory at $x = 0$, and sells it along the distribution route $x > 0$. The selling price of the product, p , is a function of the length of time after it was produced, t , and the location at which it is sold, x , i.e. $p = p(t, x)$. At a given location the price decreases in time at a rate of $-8\$/\text{hr}$. In addition, because of shipping costs, the price increases with distance from the factory at a rate of $0.1\$/\text{km}$. If the manufacturer wants to sell the product at the same price of $\$100$ everywhere, determine how fast he must travel along the distribution route. [10]
- (b) The governing equation for a fully developed steady laminar flow of a Newtonian viscous fluid on an inclined flat surface is given by: [10]

$$\mu \frac{d^2v}{dx^2} + \rho g \cos \theta = 0$$

where

$\mu = \text{coefficient of viscosity}$

$v = \text{fluid velocity}$

$\rho = \text{density}$

$g = \text{acceleration due to gravity}$

$\theta = \text{angle of the inclined surface with the vertical}$

The boundary conditions are given by:

$$\left(\frac{dv}{dx} \right)_{x=0} = 0$$

$$v(w) = 0$$

Here w is the thickness of the laminar film, and x is measured from the free surface of the film. Find the velocity distribution $v(x)$ using the weighted residual method.

5. (a) Consider the following function: [10]

$$y(x) = 1 + \cos(0.1\pi x) + \sin(0.1\pi x)$$

Use quadratic interpolation to approximate the given function, at three points, within the domain $0 \leq x \leq 1$, and calculate the error.

- (b) Consider the cooling of a circular fin by convective heat transfer along its length. [10] The cylindrical fin has a uniform cross-sectional area A , and perimeter P . The base of the fin is at a temperature of $T_B = 300^\circ\text{C}$, and the free end is insulated. The fin is exposed to an ambient temperature $T_A = 30^\circ\text{C}$. Calculate the temperature distribution along the fin, using a trial function:

$$T(x) = c_0 + c_1x + c_2x^2$$

Data:

$$n^2 = \frac{hP}{kA} = 25 \text{ m}^{-2} \quad L = 1 \text{ m}$$

6. Consider the steady two-dimensional heat conduction in a metal slab of size $2\text{m} \times 2\text{m}$. [20]
The boundary conditions are given by:

$$T(x,0) = 20^\circ\text{C}, \quad T(0,y) = 20^\circ\text{C}, \quad T(2,y) = 20^\circ\text{C}, \quad \text{and} \quad T(x,2) = 400^\circ\text{C}$$

Using five equally spaced grid points on each side, and ADI scheme, calculate the distribution of $T(x,y)$ across the metal slab.
