Computational Fluid Dynamics

Chemical VI CBSGS CFD 7/6/2016 **Q. P. Code : 734000**

(REVISED COURSE)

(3 Hours) **[Total Marks: 80**

N.B. :

1) **Question – 1 is compulsory**. Answer any **three** questions from remaining.
2) Assume data if necessary and **specify the assumptions** clearly.
3) Draw neat sketches wherever required.
4) Answer to the sub-questions of an individual question should be grouped and written together i.e. one below the other.

1. (a) Write a brief note on the applications of CFD in chemical engineering with examples. [05]

(b) Consider the following function: [05]

$$f(x,y) = e^{-x^2} - 2e^{-y^2}$$

Calculate $\frac{\partial f}{\partial y}$ at $x = 0.2$, $y = 0.2$ using the Backward difference formula. Take $\Delta x = 0.01$, $\Delta y = 0.01$. Calculate the percentage error.

(c) Derive the implicit numerical scheme to solve the following partial differential equation: [05]

$$\frac{\partial u}{\partial t} = 5\frac{\partial^2 u}{\partial x^2} + 10\frac{\partial^2 u}{\partial y^2}$$

(d) Calculate the linear approximation of $\sin(x)$, in the domain $0 \le x \le \pi/2$. [05]

2. (a) The governing equation for a fully developed steady laminar flow of a Newtonian viscous fluid on an inclined flat surface is given by: [10]

$$\mu\frac{d^2v}{dx^2} + \rho g\cos\theta = 0$$

where

$\mu = coefficient\ of\ viscosity$
$v = fluid\ velocity$
$\rho = density$
$g = acceleration\ due\ to\ gravity$
$\theta = angle\ of\ the\ inclined\ surface\ with\ the\ vertical$

The boundary conditions are given by:

$$\left(\frac{dv}{dx}\right)_{x=0} = 0$$

$$v(L) = 0$$

Find the velocity distribution $v(x)$ using the weighted residual method.

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(b) Consider the following equation: [10]

$$\frac{d^2y}{dx^2} - y(x) = 0 \qquad 0 \le x \le 1$$

The given B.C. are: $at\ x = 0,\ y = 5,\ and\ at\ x = 1.0,\ \frac{dy}{dx} = 0.$
Using an appropriate trial function, and applying the Weak Form of Galerkin approach, determine the solution.

3. Consider a cylindrical aluminium fin, being used to enhance the heat transfer from a wall, which is at a temperature of 300^oC. The diameter of the fin is $1\,mm$, and its length is $50\,mm$. The governing equation is given by: [20]

$$k\frac{d^2T}{dx^2} = \frac{Ph}{A}(T - T_A)$$

where

$$k = thermal\ conductivity$$
$$P = perimeter$$
$$A = cross - sectionalarea$$
$$h = heat - transfer\ coefficient$$
$$T_A = ambient\ temperature$$
$$T_w = wall\ temperature$$

The boundary conditions are given by:

$$T(x = 0) = T_w = 300\,^oC \quad and \quad \left(\frac{dT}{dx}\right)_{x=L} = 0$$

Obtain the temperature distribution, along the fin, using a trial function:

$$T(x) = c_0 + c_1x + c_2x^2$$

4. Consider a large plate of thickness $L = 2\,cm$ with constant thermal conductivity $k = 0.5\,W/m.K$ and uniform heat generation $q = 1000\,kW/m^3$. The faces A and B are at temperatures of $100\,^oC$ and $200\,^oC$ respectively. Assuming that the dimensions in the y *and* z directions are so large that temperature gradients are significant in the $x-direction$ only, calculate the temperature distribution, assuming five control volumes. [20]

5. Consider the steady heat conduction in two dimensions: [20]

$$\frac{\partial^2T}{\partial x^2} + \frac{\partial^2T}{\partial y^2} = 0$$

The boundary conditions are given by:

$$T(x, y)\ at\ x = 0\ is\ 40^oC, \quad and \quad T(x, y)\ at\ x = 2m\ is\ 60^oC$$

$$T(x, y)\ at\ y = 0\ is\ 40^oC, \quad and \quad T(x, y)\ at\ y = 2m\ is\ 200^oC$$

Obtain the temperature profile $T(x, y)$, considering five nodes in each direction, and using the ADI method.

6. (a) Diffusion and reaction take place in a pore of length 1 mm. The rate constant of the first order reaction is $k = 10^{-3}\, s^{-1}$, and the effective diffusivity of the reacting species is $D = 10^{-9}\, m^2/s$. Dividing the pore into five equal parts obtain the concentration profile along its lenfth, using central differencing scheme. The concentration at the mouth of the pore is $C(0) = 1\, mol/m^3$. The governing equation is given by: [10]

$$\frac{d^2C}{dx^2} - \frac{k}{D}C = 0$$

with the boundary conditions:

$$C(0) = 1 \quad and \quad at\ x = 1\, mm, \frac{dC}{dx} = 0$$

(b) Explain the use of Upwind scheme with a suitable example. [10]

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