

(Time: - 3 hrs)

Maximum Marks: - 80

N. B.

1. Question no. 1 is compulsory.
2. Answer any three out of the remaining five questions.
3. Assumption made should be clearly stated.
4. Assume any suitable data wherever required but justify the same.
5. Illustrate the answers with sketches wherever required.
6. Answer to the questions should be grouped and written together.

Q1 Write the short notes on: 20

- a. Principal of Minimum Potential Energy
- b. Explain the significance of Jacobian matrix.
- c. Types of Boundary Conditions
- d. Sources of error in FEM

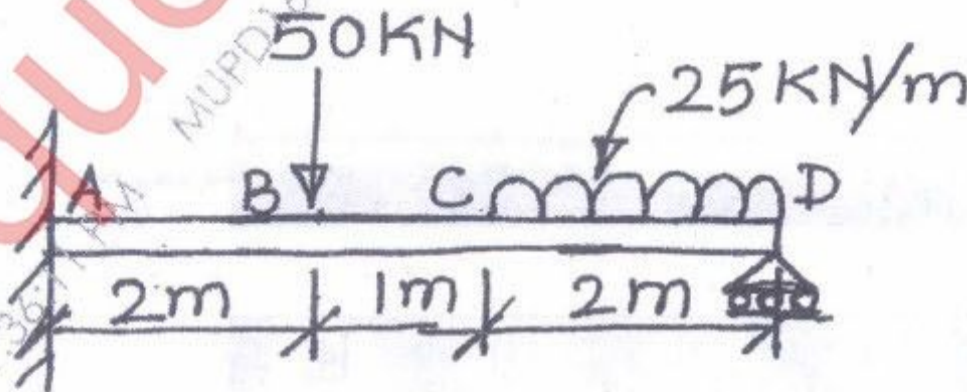
Q2 a. Solve the following differential Equation using Galerkin Method. 10

$$\frac{d^2 \phi}{dx^2} + \cos \pi x = 0 \quad 0 < x < 1.$$

Boundary Conditions are: $\phi(0)=0$, $\phi(1)=0$

Find $\phi(0.25)$, $\phi(0.5)$ and $\phi(0.75)$. Compare your answer with exact solution.

- b. Figure shows the beam of uniform rectangular cross section 10 cm x 12 cm, subjected to point load and uniformly distributed load. Young's modulus is 2 MPa and Poisson's ratio is 0.3. Determine the deflections and slopes. 10



TURN OVER

- Q3 a. Consider the steady laminar flow of a viscous fluid through a long circular cylindrical tube. The governing equation is 15

$$-\frac{1}{r} \frac{d}{dr} \left(r \mu \frac{dw}{dr} \right) = \frac{P_0 - P_L}{L} = f_0$$

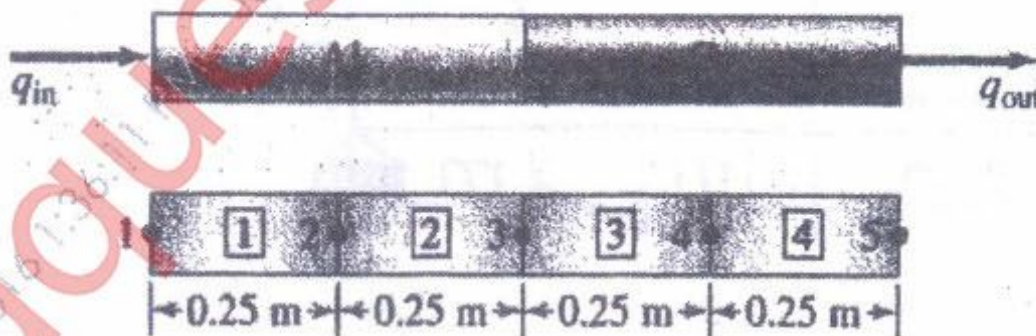
where w is the axial (i.e., z) component of velocity, μ is the viscosity, and f_0 is the gradient of pressure (which includes the combined effect of static pressure and gravitational force). The boundary conditions are

$$\left(r \frac{dw}{dr} \right) \Big|_{r=0} = 0, \quad w(R_0) = 0$$

Using the symmetry and two linear elements, determine the velocity field and compare with the exact solution at the nodes:

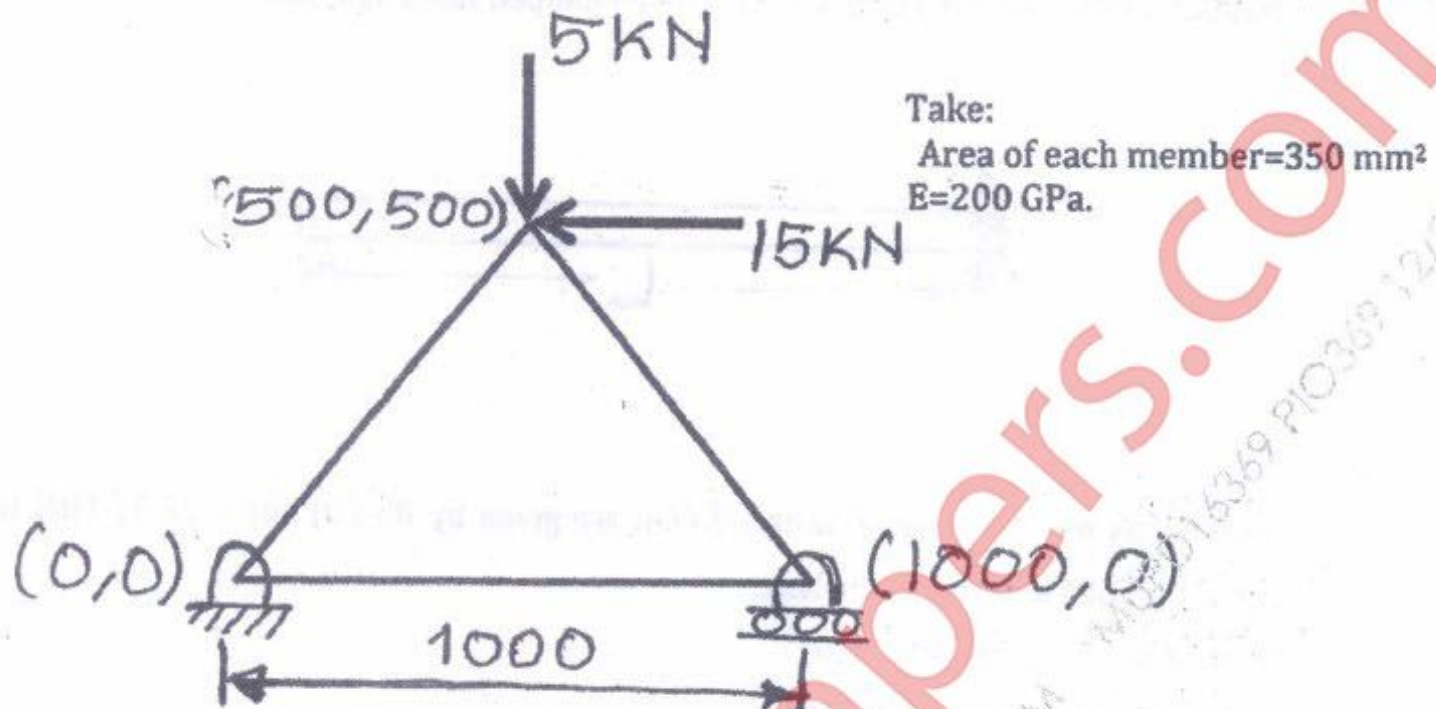
$$w(r) = \frac{f_0 R_0^2}{4\mu} \left[1 - \left(\frac{r}{R_0} \right)^2 \right]$$

- b. Derive shape function for 1D linear element in natural co-ordinates 5
- Q4 a. The circular rod depicted in Figure has an outside diameter of 60 mm, length of 1 m, and is perfectly insulated on its circumference. The left half of the cylinder is aluminum, for which $k_x = 200 \text{ W/m}^\circ\text{C}$ and the right half is copper having $k_x = 389 \text{ W/m}^\circ\text{C}$. The extreme right end of the cylinder is maintained at a temperature of 80°C , while the left end is subjected to a heat input rate 4000 W/m^2 . Using four equal-length elements, determine the steady-state temperature distribution in the cylinder. 10

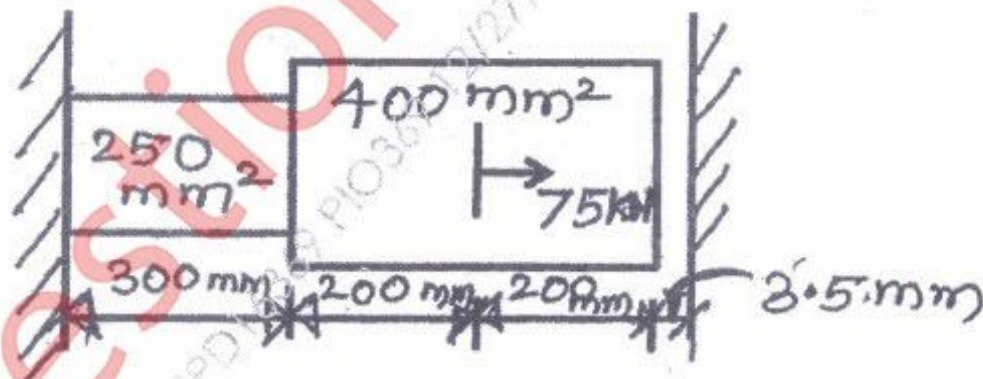


TURN OVER

- b. Analyze the truss completely for displacement, stress and strain as shown in figure. 10



- Q5 a. Find nodal displacement and element stress for the bar as shown in figure using FEM. 10
Take E = 200 GPa.



- b. A CST element has nodal coordinates (10,10), (70,35) and (75,25) for nodes 1, 2 and 3 respectively. The element is 2 mm thick and is of material with properties E = 70 GPa. Poisson's ratio is 0.3. After applying the load to the element the nodal deformation were found to be $u_1 = 0.01$ mm, $v_1 = -0.04$ mm, $u_2 = 0.03$ mm, $v_2 = 0.02$ mm, $u_3 = -0.02$ mm, $v_3 = -0.04$ mm. Determine the strains e_x , e_y , e_{xy} and corresponding element stresses

- Q6 a. Consider a uniform cross section bar of length L made up of a material whose Young's modulus and density are given by E and ρ . Estimate the natural frequencies of axial vibration of the bar using both consistent and lumped mass matrices. 10



- b. 10
 Coordinates of the nodes of finite element are given by $P(4,0)$ and $Q(8,0)$. Find the expression of x in terms of ξ when:
- 1) Third node R is taken at $(6,0)$
 - 2) Third node R is taken at $(5,0)$
- Comment on the result.
