Sy BSc IT Sem III Reg Exam Oct 18

31/10/18

Paper / Subject Code: 80705 / Applied Mathematics

(Time: 2½ hours)

Total Marks: 75

N. B.: (1) All questions are compulsory.
(2) Make suitable assumptions wherever necessary and state the assumptions made.
(3) Answers to the same question must be written together.
(4) Numbers to the right indicate marks.
(5) Draw neat labeled diagrams wherever necessary.
(6) Use of Non-programmable calculators is allowed.

1. Attempt *any three* of the following: 15

a. Reduce the matrix to normal form and find its rank where

$$A=\begin{bmatrix}1 & -1 & 3 & 6\\ 1 & 3 & -3 & -4\\ 5 & 3 & 3 & 11\end{bmatrix}$$

b. Examine for consistency the system of equations $x-y-z=2;\quad x+2y+z=2;\quad 4x-7y-5z=2$ and solve them if found consistence.

c. Verify Cayley – Hamilton Theorem for the matrix A.

$$A=\begin{bmatrix}1 & 2 & -2\\ -1 & 3 & 0\\ 0 & -2 & 1\end{bmatrix}$$

d. Express in Polar from $-1+\sqrt{3}i$

e. Simplify $\dfrac{(\cos\theta-\sin\theta)^6(\cos 5\theta-i\sin 5\theta)^{-2}}{(\cos 8\theta+i\sin 8\theta)^{1/2}}$ using De-Moivre's theorem.

f. Prove that : $\sinh^{-1}x=\log\left(x+\sqrt{x^2+1}\right)$

2. Attempt *any three* of the following: 15

a. Solve $y^2-x^2\dfrac{dy}{dx}=xy\dfrac{dy}{dx}$

b. Solve $\dfrac{dy}{dx}+2y\tan x=\sin x$

c. Solve $(p-2x)(p-y)=0$

d. Solve : $y=xp+\dfrac{1}{p}$

e. Solve : $(D^2+6D+9)y=5^x-\log 2$

f. Solve : $x^2\dfrac{d^2y}{dx^2}-x\dfrac{dy}{dx}-3y=0$

3. Attempt *any three* of the following: 15

a. Find the Laplace transform of $f(t)=\begin{cases}\cos t & 0<t<\pi\\ \sin t & t>\pi\end{cases}$

b. Evaluate by using Laplace transform $\int_0^\infty t^2 e^{-t}\sin t\,dt$

c. Find the Laplace transform of the following.

$$\frac{dy}{dt}+3y(t)+2\int_0^t y(t)dt=t; \quad given\ y(0)=0$$

d. Find the inverse Laplace transform of $\frac{s}{(s-2)^4}$

e. Find inverse Laplace transform of $\cot^{-1}(s)$

f. Find the Laplace transform of $f(t)=\begin{cases}1 & 0<t<a\\ -1 & a<t<2a\end{cases}$ and $f(t)=f(t+2a)$

4. Attempt *any three* of the following: 15

a. Evaluate : $\int_0^1\int_0^y xy\,e^{-x^2}dx\,dy$

b. Take Expression as a single integral and evaluate

$$\int_0^{a/\sqrt{2}}\int_0^x x\,dx\,dy+\int_{a/\sqrt{2}}^a\int_0^{\sqrt{a^2-x^2}} x\,dx\,dy$$

c. Evaluate $\int_0^a\int_0^{\sqrt{a^2-y^2}}\left(\sqrt{a^2-x^2-y^2}\right)dx\,dy$

d. Evaluate : $\iiint_V \frac{dx\,dy\,dz}{(x+y+z+1)^3}$ where V is the volume bounded by the planes,

$x=0,\ y=0,\ z=0, and\ x+y+z=1.$

e. Evaluate $\iint xy(x+y)\,dx\,dy$ over the area between curve $y=x^2$ and the line $y=x$

f. Prove that the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{e^2}=1$ is $\frac{4\pi}{3}abc$

[TURN OVER]

5. **Attempt *any three* of the following:** 15

a. Evaluate $\int_0^{\infty} x^2 . e^{-h^2x^2} . dx$

b. Evaluate $\int_0^{\pi} x \sin^6 x \, dx$

c. Show that : $\int_0^{\pi/2} \frac{\log(1 + a\sin^2 x)}{\sin^2 x} . dx = \pi\left[\sqrt{1+a} - 1\right]$

d. Show that : $\int_0^{\infty} \frac{\sin x}{x} . dx = \frac{\pi}{2}$

e. Find : $\frac{d}{dx}\left[erf(x) + erf_c(ax)\right]$

f. If $\phi(\alpha) = \int_{f(\alpha)}^{g(\alpha)} F(x, \alpha) \, dx$, write the rule to find $\frac{d\phi}{d\alpha}$ and hence prove that,

$$\frac{d}{dx}\left[erf\sqrt{x}\right] = \frac{e^{-x}}{\sqrt{\pi x}}$$