

(Time: 2½ hours)

Total Marks: 75

- N. B.: (1) All questions are compulsory.
 (2) Make suitable assumptions wherever necessary and state the assumptions made.
 (3) Answers to the same question must be written together.
 (4) Numbers to the right indicate marks.
 (5) Draw neat labeled diagrams wherever necessary.
 (6) Use of Non-programmable calculators is allowed.

1. Attempt any three of the following:

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- a. Find the Adjoint of the given matrix and hence find Inverse if exist

$$\begin{bmatrix} 2 & -1 & 3 \\ 4 & 6 & -2 \\ 5 & 1 & 8 \end{bmatrix}$$

- b. Find the Characteristic values and characteristic vectors of the given matrix.

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

- c. Verify Caley-Hamilton theorem for the given matrix, also find inverse if exists.

$$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

- d. Expand $(1 + \cos x + i \sin x)^n$
 e. Evaluate $(1 + i\sqrt{3})^{16} / (\sqrt{3} - i)^{17}$
 f. Express $\sec(x + iy)$ in $a + ib$ form

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2. Attempt any three of the following:

- a. Solve the Differential Equation $(x - 4xy - 2y^2) dx + (y^2 - 4xy - 2x^2) dy = 0$
 b. Solve the Differential Equation $dy/dx + x^2y = x^5$
 c. Solve the following Equation $x^2p^2 - 2xpy + (2y^2 - x^2) = 0$
 d. Solve the following Equation $p(p + y) = x(x + y)$
 e. Find the Complementary and Particular Solution of the equation $(D^3 + D^2 + D + 1)y = \sin 2x$
 f. Find the General Solution of the equation $(D^2 + 4)y = \sin 3x + e^x + x^2$

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3. Attempt any three of the following:

- a. Evaluate $\int_0^{\infty} e^{-2t} \sin^2 t dt$
 b. Find the inverse Laplace transform for the function

$$F(s) = \frac{21-s^2}{s(s^2+4s+13)}$$

 c. Find Laplace transformation of the function

$$f(t) = te^{2t} \cos 3t$$

[TURN OVER]

- d. Obtain the Inverse Laplace transform of each of the given function

$$\frac{(s+1)}{s^3(s-3)^2}$$

- e. Find Inverse Laplace Transformation by convolution theorem for

$$F(s) = \frac{s}{(s^2+1)(s^2+4)}$$

- f. By using fundamental definition, find laplace transform of $f(t)$

$$F(t) = t, \quad 0 < t < 4$$

$$= 5, \quad t > 4$$

4. Attempt **any three** of the following:

a. Evaluate $\int_0^1 \int_0^2 e^{x+y} dx dy$

b. Evaluate $\int_0^3 \int_0^{\sqrt{4-y}} \frac{dx dy}{(1+x^2+y^2)}$

c. Evaluate $\int_0^{\log 2} \int_0^x \int_0^{x+\log y} e^{x+y+z} dx dy dz$

d. Evaluate $\int_0^1 \int_0^{1-x} \int_0^{x+y} e^z dx dy dz$

e. Change the order of integration and evaluate $\int_0^2 \int_0^{x^2/4} xy dx dy$

f. Solve $\iint r^3 dr d\theta$ over the area included between the circles $r = 2\sin\theta$ and $r = 4\sin\theta$

5. Attempt **any three** of the following:

a. Evaluate $\int_0^{\pi/2} \sin^6 x \cos^7 x dx$

b. Evaluate i) $\operatorname{erfc}(-x) + \operatorname{erfc}(x)$
ii) $\operatorname{erfc}(x) + \operatorname{erf}(x)$

c. Evaluate $\int_0^{2a} x(2ax - x^2)^{1/2} dx$

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d. Evaluate $\int_0^{\pi/2} \sin^5 2x dx$

e.

Evaluate $\int_0^1 \frac{x^7}{(1-x^4)^{1/2}} dx$

f.

Evaluate $\int_0^1 \frac{(x^a - x^b)}{\log x} dx$