S4CS (1)

(2 ½ Hours) [Total Marks: 75]

N.B. 1) All questions are compulsory.
2) Figures to the right indicate marks.
3) Illustrations, in-depth answers and diagrams will be appreciated.
4) Mixing of sub-questions is not allowed.

Q. 1 Attempt All (Each of 5Marks) **(15M)**

(a) **Select correct answer from the following:**

1. The product of two consecutive natural number is always divisible by_____.
   (a) 3 (b) 2 (c) 6 (d) 10
2. The value of ${}^{7}P_3$ =_____.
   (a) 35 (b) 210 (c) 30 (d) 7!/3!
3. A vertex with degree one is called _____ vertex.
   (a) Pendant (b) isolated (c) incident (d) none of the above
4. A graph with parallel edges and loops is called a _____ graph.
   (a) simple (b) pseudo (c) multiple (d) none of the above
5. The out- degree of Sink 'z' of a Network is_____ .
   (a) zero (b) No. of vertices (c) 1 (d) none of the above

(b) **Fill in the blanks**

(Coefficients, Chromatic , n, degree, equal, one)

1. The Pascal triangle is used to find the _____ in binomial expansion.
2. ${}^{n}C_n$ =_____
3. The number of edges incident on a vertex is called _____ of vertex
4. Minimum number of colours required to colour the vertices of the graph is called _____number of the graph.
5. In network the amount of flow leaving the source is _____ to the amount of flow arriving at the sink.

(c) **Short Answers**

1. Second Principal of Mathematical Induction
2. Binomial theorem
3. Labeled Tree
4. Planar graph
5. Flow

Q. 2 Attempt the following (Any THREE) **(15M)**

(a) How many license plates can be made using either two letters followed by four digits or two digits followed by four letters?

(b) Determine the coefficient on $x^2y^3z^2$ in the expansion of $(x + y + z)^7$.

(c) For any positive integer n, the sum of squares of the first n positive integers is $\frac{n(n+1)(2n+1)}{6}$, Prove by first principle of mathematical induction.

(d) How many integer- valued solutions are there for the equation
$x_1 + x_2 + x_3 + x_4 + x_5 = 65$, all $x_i \geq 0$

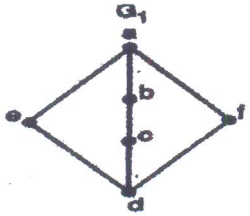
(e) What is Sudoku Puzzles ? Write its benefits

(f) For each n >0 , prove that

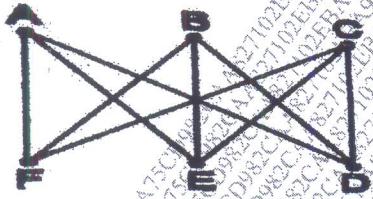
$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \cdots + (-1)^n \binom{n}{n} = 0$$

Q. 3 Attempt the following (Any THREE) **(15M)**

(a) Check whether the following graphs are isomorphic or not.



(b) Verify Euler's formula for the given connected graph.



(c) What is bipartite graph? Show C_6(cycle of six vertices) is a bipartite graph.

(d) State Ramsey's theorem for graphs and also estimate Ramsey Numbers R(2,4) and R(3,5)

(e) Define adjacency matrix representation of a graph also draw the graph for the given adjacency matrix.

$$\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

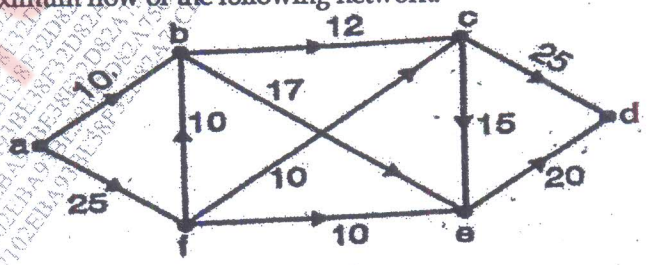
(f) Give an example of graph which is both Eulerian and Hamiltonian and justify it.

Q. 4 Attempt the following (Any THREE) **(15)**

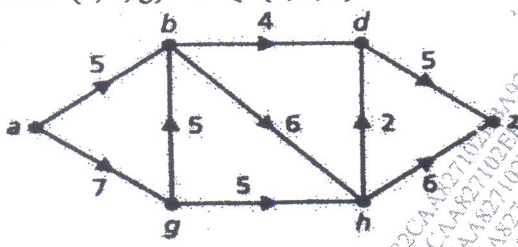
a) Explain Polya's enumeration formula using chain index.

(b) Explain Burnside's Lemma

(c) Find maximum flow of the following network.



(d) Define the capacity of cuts. Find the capacity of the cut (P,Q), where P={a, b, g} and Q={d, h, z}



(e) Write permutations shown below in cycle notation , compute $\pi_1\pi_2$ (product of two permutations) and inverse of π_1.

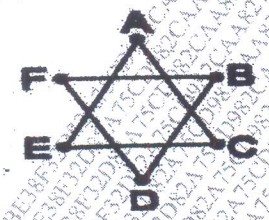
$\pi_1 = \begin{pmatrix} 1 2 3 4 5 6 7 8 \\ 3 1 5 8 2 6 4 7 \end{pmatrix}, \pi_2 = \begin{pmatrix} 1 2 3 4 5 6 7 8 \\ 3 7 1 6 8 4 2 5 \end{pmatrix}$

(f) Explain a Complete matching with example.

Q. 5 Attempt the following (Any THREE) **(15)**

(a) In how many ways we can arrange the letters in the word MATHEMATICS?

(b) Find Chromatic number and Clique of the given graph.



(c) Explain the integer solutions of linear programming problems.

(d) Determine the union and intersection of the graphs G_1 and G_2.

(e) **Draw all regular graphs on 4 vertices with degree two.**

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