

(3hours)



[Total marks: 80]

- N.B.** (1) Question No. 1 is compulsory.  
 (2) Answer **any Three** from remaining  
 (3) Figures to the right indicate full marks

1. (a) State Cauchy Reimann equation in polar form. Find p if

$$f(z) = r^2 \cos 2\theta + ir^2 \sin p\theta \text{ is analytic.} \quad 5$$

(b) Find Laplace transform of  $\sin 2t \cdot \cos 3t$ . 5

(c) Prove  $\{\sin nx\}$ ,  $n = 1, 2, 3, \dots$  is orthogonal w.r.t.  $(0, 2\pi)$ . 5

(d) Evaluate  $\int_{1+i}^{2+4i} (x^2 + ixy) dz$  along the curve  $x = t, y = t^2$ . 5

2. (a) Using Laplace transform, solve the differential equation,

$$\frac{dx}{dt} + 3x = 2 + e^{-t}, \text{ with } x(0) = 1. \quad 6$$

(b) Evaluate  $\oint_C \frac{z+1}{z^3 - 2z^2} dz$  where  $C: |z| = 1$ . 6

(c) Obtain the Taylor's and Laurent series which represent the function  $\frac{z^2 - 1}{(z+3)(z+4)}$  in the regions, (i)  $|z| < 3$  (ii)  $3 < |z| < 4$  (iii)  $|z| > 4$ . 8

3. (a) Solve  $\frac{\partial^2 u}{\partial x^2} - 32 \frac{\partial u}{\partial t} = 0$  by Bender-Schmidt method, given  $u(0, t) = u(x, 0) = 0, u(1, t) = t$ , taking  $h = 0.25$ . 6

(b) Evaluate  $\int_0^{\infty} t e^{-3t} \sin t dt$  6

(c) Obtain Half Range Sine Series of  $f(x) = x(\pi - x)$  in  $(0, \pi)$ .

Hence, evaluate  $-\sum_{m=0}^{\infty} \frac{(-1)^m}{(2m+1)^3}$ . 8

[TURN OVER]

4. (a) Find the orthogonal trajectory of the family of curves  $x^3 y - xy^3 = 0$ . 6

(b) Find Fourier series of  $f(x) = |x|$  in  $(-3, 3)$ . 6

(c) Find the inverse Laplace transform of the following:-

(i)  $\cot^{-1} s$                       (ii)  $\frac{8e^{-3s}}{s^2 + 4}$  8

5. (a) Solve by Crank –Nicholson simplified formula  $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$ ,  
 $u(0, t) = u(1, t) = 0$ ,  $u(x, 0) = 100x(1 - x)$   
 taking  $h = 0.25$  for one time step. 6

(b) Using convolution theorem find the inverse Laplace transform of  

$$\frac{s}{(s^2 + 1)(s^2 + 4)}$$
 6

(c) Find bilinear transformation which maps the points  $z = 1, i, -1$  onto the points  $w = i, 0, -1$ . Hence, find the image of  $|z| \leq 1$  onto the  $w$ -plane. 8

6. (a) Using Residue theorem, evaluate,  $\int_0^{\infty} \frac{dx}{x^2 + 1}$ . 6

(b) Obtain Complex form of Fourier series for  $f(x) = e^{ax}$  over  $-\pi < x < \pi$ . 6

(c) Determine the solution of one-dimensional heat equation  $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$   
 under boundary condition  $u(0, t) = u(l, t) = 0$ ,  $u(x, 0) = x$ ,  $l$  being the length of rod. 8