sem-III / App. Maths-III / MECH & PROD

31-05-14

QP Code : NP-18610

(3 Hours) **[Total Marks :80**

N.B. (1) Question no. **1** is **compulsory.**
(2) Solve any **three** questions out of the remaining Q.no. **2** to Q. no. **6.**

1. (a) Find the inverse Laplace transform of **20**

$$\frac{S^2+5}{(S^2+4S+13)^2}$$

(b) If $V = 3x^2y + 6xy - y^3$, show that the function V is harmonic, find the corresponding analytic function.

(c) Evaluate $\int_C \bar{z}dz$ where C is the upper half of the circle $r = 1$.

(d) Prove that $f_1(x) = 1$, $f_2(x) = x$, $f_3(x) = \frac{3x^2-1}{2}$ are orthogonal over $(-1, 1)$.

2. (a) Evaluate $\int_0^\infty \frac{\cos at - \cos bt}{t} dt$ **6**

(b) Obtain complex form of fourier series $f(x) = e^{-ax}$ for in $(-\pi, \pi)$ **6**

(c) Using Crank-Nicholson simplified formula solve, $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$ **8**

$u(0,t) = 0, u(4,t) = 0, u(x,0) = \frac{x}{3}(16 - x^2)$

Find uij for $i = 0, 1, 2, 3, 4$, and $j = 0, 1, 2$.

3. (a) Evaluate $\int_C \frac{\sin^6 z}{(z - \pi/6)^3} dz$ where C is $|z| = 1$ **6**

(b) Find the fourier expansion for $f(x) = x - x^2 \; -1 < x < 1$ **6**

(c) Determine the solution of one dimensional heat equation, $\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$ under the boundary conditions $u(0, t) = 0$ $u(\ell, t) = 0$ and $u(x, 0) = x$, $(0 < x < \ell)$, ℓ being length of the rod. **8**

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4. (a) Find inverse Laplace transform by using convolution theroem, 6

$$f(s)=\frac{s^2}{(s^2-a^2)^2}$$

(b) Find the image of the region bounded by $x=0, x=2, y=0, y=2$ in the Z plane under transformation $W=(1+i)Z$. 6

(c) Find all possible Laurent's expansions of the function $f(z)=\frac{7z-2}{z(z-2)(z+1)}$ about $Z=-1$. 8

5. (a) Solve $\frac{\partial^2 u}{\partial x^2}-32\ \frac{\partial u}{\partial t}=0$ by Bender-Schmidt method, subject to the conditions $u(0,t)=0, u(x,0)=0, u(1,t)=t$ taking $h=0.25, 0<x<1$. 6

(b) Obtain hlaf range sine series for f(x) when 6

$$f(x)=x, \quad 0<x<\frac{\pi}{2}$$

$$=\pi-x, \quad \frac{\pi}{2}<x<\pi$$

(c) Evaluate $\int_{-\infty}^{\infty}\frac{x^2dx}{(x^2+a^2)(x^2+b^2)}$ by using residues. $a>0, b>0$ 8

6. (a) Find the orthogonal tranjectory of the family of curves $x^3y-xy^3=c$. 6

(b) Obtain the fourier expansion of $f(x)=\left(\frac{\pi-x}{2}\right)^2$ in the interval 6

$0<x<2\pi, \quad f(x+2\pi)=f(x)$

Also deduce that $\frac{\pi^2}{6}=\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots\ldots\ldots$

(c) Solve using Laplace transform $(D^2-3D+2)y=4e^{2t}$, with $y(0)=-3\ y'(0)=5$. 8
