S.E. (SEM. IV) (CBSGS) (MECHANICAL ENGG.) 15th May, 2015

APPLIED MATHEMATICS - IV

QP Code : **3451**

Rev Course

(3 hours) [Total Marks: 100

1.Q1 is compulsory

2. Solve any three out of the remaining from Q.2 to Q. 6.

3. Figures on the right hand side indicate marks.

4. Use of statistical tables is allowed.

Q.1. a) A continuous random variable with P.D.F. $f(x) = k\,x(1-x)$, $0 \le x \le 1$. Find K and determine a number b such that $P(x \le b) = p(x \ge b)$. 5

b) If $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$, Find the characteristic roots of A and $A^3 + I$. 5

c) By using Green's theorem Show that the area bounded by a simple closed curve c is given by $\frac{1}{2}\int_C xdy - ydx$ 5

d) If the tangent of the angle made by the line of regression of y on x is 0.6 and $\sigma_y = 2\sigma_x$. Find the correlation coefficient between x and y. 5

Q. 2. a) The means of two random samples of size 9 and 7 are 196.42 and 198.82 respectively. The sum of the squares of the deviation from the means is 26.94 and 18.73 respectively. Can the sample be considered to have been drawn from the same population? 6

b) If the vector field $\bar{F}$ is irrotational, find the constants a,b,c where $\bar{F} = (x + 2y + az)\bar{i} + (bx - 3y - z)\bar{j} + ((4x + cy + 2z)\bar{k}$ Show that $\bar{F}$ can be expressed as the gradient of a scalar function. Then find the work done in moving a particle in this field from (1,2,-4) to (3,3,2) along the straight line joining the points. 6

c) Using the Kuhn Tucker conditions solve the following N.L.P.P. Maximize $Z = x_1^2 + x_2^2$, subjected to $x_1 + x_2 - 4 \le 0$ and $2x_1 + x_2 - 5 \le 0$, $x_1, x_2 \ge 0$. 8

[Turn over

Q3. a)Seven dice are thrown 729 times. How many times do you expect at least four dice to show three or five? 6

b) Evaluate by using Stokes theorem, $\int_c xydx + xy^2dy$, c is the square in xy- plane with vertices (1,0),(0,1), (-1,0)and (0,-1). 6

c) In a laboratory experiment two samples gave the following results. Test the equality of sample variances at 5% level of significance. 8

Sample	size	mean	Sum of squares of the deviations from mean
1	10	15	90
2	13	14	108

Q. 4. a)Can it be concluded that the average life span of an Indian is more than 70 years, if a random sample of 100 Indians has an average life span of 71.8 years with the Standard deviation of 7.8 years . 6

b) Use Gauss's divergence theorem to evaluate where $\iint_S \overline{N}.\bar{F}dS$, $\bar{F}=(4x\bar{\imath} -2y^2\bar{\jmath} + z^2\,\bar{k}$, and S is the region bounded by $x^2 + y^2 = 4, z = 0, z = 3$. 6

c) Using Lagrange's method of multipliers solve the NLPP, Optimize Z= $4x_1 + 8x_2 - x_1^2 - x_2^2$ subjected to $x_1 + x_2 = 4, x_1, x_2 \geq 0$ 8

Q.5. a) Show that the matrix A=$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ is diagonalizable. Find the transforming matrix and the diagonal matrix. 6

b) Calculate the Karl Pearson's coefficient of correlation for the following data. 6

x	28	45	40	38	35	33	40	32	36	33
y	23	34	33	34	30	26	28	31	36	35

[Turn over

c) The following table gives the number of accidents in a city during a week. Find whether the accidents are uniformly distributed over a week, using χ^2 test. 8

day	Sun	Mon	Tues	Wed	Thurs	Fri	Sat	Total
No of accidents	13	15	9	11	12	10	14	84

Q6.

a) Find A^{50} if $A=\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ 6

b) The monthly salary in a big organization is normally distributed with mean Rs. 3000, and standard deviation of Rs. 250. What should be the minimum salary of a worker in the organization so that the probability that he belongs to top 5% workers. 6

c) Verify green's Theorem in the plane for where $\int_C (xy + y^2)dx + x^2dy$

c is the closed curve of the region bounded by $y = x$ and $y = x^2$. 8