S.E. (SEM. IV) (CBSGS) (MECHANICAL ENGG.) 23rd Nov. 2015

APPLIED MATHEMATICS - IV 3.00 pm to 6.00 pm

Q.P. Code : 5301

(3 Hours) **[Total Marks : 80**

N.B. (i) Question no. ONE is compulsory.

(ii) Attempt any THREE questions out of the remaining questions.

(iii) Figures to right indicate full marks.

Q. 1 (a) Find the eigen values of the adjoint of the matrix 5

$$A=\begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$$

(b) There are 10 counters in a bag,6 of which are 5 rupees each while the remaining 4are of equal, but unknown value. If the expectation of drawing a single counter at random is 4 rupees, find the unknown Value. 5

(c) It is given that the mean of x and y are 5 and 10. If the line of regression of y on x is parallel to the line 20y = 9x + 40 .Estimate the value of y for x = 30. 5

(d) Find the total work done in moving a particle in the force field $\overline{F}=3xyi-5zj+10xk$ along $x=t^2+1$, $y=2t^2$, $z=t^3$ .from t= 1 to t= 2. 5

Q.2 (a) The means of two samples of sizes 1000 and 2000 respectively are 67.50 and 68.0 inches. Can the samples be regarded as drawn from The same population of S.D. 2.5 inches? 6

(b) Find the characteristic equation of the matrix A given below and Hence find the matrix represented by $A^6-6A^5+9A^4+4A^3-12A^2+2A-I$. 6

$$A=\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$$

(c) Verify Green's theorem for $\int_c \frac{1}{y}dx+\frac{1}{x}dy$ where c is the boundary of the region defined by $x=1, x=4$, $y=1$ and $y=\sqrt{x}$ 8

Q.3 (a) Tests made on breaking strength of 10 pieces of a metal wire gave the following results. 6

578,572,570,568,572,570,570,572,596 and 584 in kgs. Test if the breaking

Strength of the metal wire can be assumed to be 577kg?

(b) The probability that at any moment one telephone line out of 10 will be busy is 0.2. 6

(i) What is the probability that 5 lines are busy ?

(ii) Find the expected number of busy lines and also find the probability of this number.

(iii) What is the probability that all lines are busy?

(c) Using the Kuhn – Tucker conditions solve the following N.L.P.P. 8

Maximise $z = 2x_1 + 3x_2 - x_1^2 - 2x_2^2$

Subject to $x_1 + 3x_2 \le 6$, $5x_1 + 2x_2 \le 10$, x_1 , $x_2 \ge 0$.

Q.4 (a) Use Gauss-Divergence theorem to evaluate $\iint_s \overline{N}.\overline{F}\, ds$ 6

Where $\overline{F} = x^2 i + zj + yz\,k$ and s is the surface of the cube bounded by x=0, x=1 y=0 ,y=1 , z=0 and z=1.

(b) The probability density function of a random variable X is, 6

X= 0 , 1 , 2 , 3 , 4 , 5 , 6 .

P(X=x) = k , 3k , 5k , 7k , 9k , 11k , 13k . Find $P(X < 4)$, $P(3 < X \le 6)$.

(c) In a test given to two groups of students drawn from two normal Populations marks obtained were as follows, 8

Group A : 18,20,36,50,49,36,34,49,41.

Group B : 29,28,26,35,30,44,46. Examine the equality of variances at 5% level of significance.

Q.5 (a) A die was thrown 132 times and the following frequencies were Observed , 6

No. obtained	: 1 , 2 , 3 , 4 , 5 , 6 .	total
Frequencies	: 15, 20, 25, 15, 29, 28.	132

Test the Hypothesis that the die is unbiased.

(b) Using the method of Lagrange's multipliers solve the given N.L.P.P. 6

Optimize $z = 6x_1^2 + 5x_2^2$,

Subject to : $x_1 + 5x_2 = 7$, $x_1, x_2 \geq 0$.

(c) Evaluate $\iint_s (\nabla \times \overline{F}).d\overline{s}$ where $\overline{F} = (2x - y + z)i + (x + y - z^2)j + (3x - 2y + 4z)k$ 8

and s is the surface of the cylinder $x^2 + y^2 = 4$ bounded by the plane $z=9$ and open at the other end.

Q.6 (a) For a normal variate with mean 2.5 and S.D. 3.5, find the probability That (i) $(2 \leq x \leq 4.5)$ (ii) $(-1.5 \leq x \leq 5.5)$ 6

(b) Soil temperature (x) and germination interval(y) for winter wheat in 12 Places are as follows- 6

x(in ^0F) : 57,42,38,42,45,42,44,40,46,44,43,40

y(days) : 10,26,41,29,27,27,19,18,19,31,29,33

(c) Find e^A and 4^A if $A = \begin{bmatrix} \frac{3}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{3}{2} \end{bmatrix}$ 8
