Q. P. Code: 25569

(3hours) [Total marks: 80]

N.B. 1) Question No. 1 is compulsory.
2) Answer **any Three** from remaining
3) Figures to the right indicate full marks

1. a) State Cauchy Reimann equation in polar form. Use them to find p if $f(z) = r^2 \cos 2\theta + i \sin p\theta$ is analytic. 5

b) Find Laplace transform of $f(t) = te^{-3t} \sin t$. 5

c) Find half-range sine series for $f(x) = \frac{\pi}{4}$ in $(0, \pi)$. Hence, show that 5

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

d) Evaluate $\int_c (z - z^3)\, dz$, where C is left half of the unit circle from $-i$ to i. 5

2. a) Obtain the Taylor's and the Laurent series which represent the function $f(z) = \frac{2}{(z-1)(z-2)}$ in the regions, i) $|z| < 1$ ii) $1 < |z| < 2$ 6

b) Obtain complex form of Fourier series of $f(x) = e^{-x}$, $-1 < x < 1$ in $(-1, 1)$. 6

c) Using Laplace transform, solve the differential equation, $\frac{dx}{dt} + 2x = \cos\omega t$, with $x(0) = 0$. 8

3. a) Solve $\frac{\partial^2 u}{\partial x^2} - 100\frac{\partial u}{\partial t} = 0$ with $u(0,t) = 0, u(1,t) = 0, u(x,0) = x(1-x)$ taking $h = 0.1$ for three time steps up to $t = 1.5$ by Bender –Schmidt method. 6

b) Find the bilinear transformation which maps the points $z = 0, -1, i$ into the points $w = i, 0, \infty$. 6

c) Obtain Fourier Series of $f(x) = \begin{cases} x, & 0 < x \le \pi \\ 2\pi - x, & \pi \le x < 2\pi \end{cases}$ in $(0, 2\pi)$ 8

Hence, deduce that –

$$\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots$$

[TURN OVER]

4. a) Find the orthogonal trajectory of the family of curves $2x - x^3 + 3xy^2 = c$ 6
   b) Find the Fourier series for $f(x) = 1 - x^2$ in $(-1, 1)$. 6

   c) Find the inverse Laplace transform of:
   i) $F(s) = \frac{1}{s(s^2+9)}$, using Convolution theorem, ii) $F(s) = cot^{-1}(s+1)$. 8

5. a) Solve by Crank –Nicholson simplified formula $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$,

   $u(0,t) = 0,\ u(5,t) = 100,\ u(x,0) = 20$ taking $h = 1$ for one-time step. 6

   b) Find the image of the circle $|z| = 4$ in the z-plane under the transformation $w = z + 2 + 3i$. Draw the sketch. 6

   c) If $v = 3x^2y + 6xy - y^3$, show that v is harmonic and find the corresponding analytic function $f(z) = u + iv$. 8

6. a) Using Residue theorem, evaluate, $\int_0^{2\pi} \frac{d\theta}{5 - 3\cos\theta}$ 6

   b) Using Laplace transform, evaluate $\int_0^\infty e^{-t}(1 + 3t + t^2)H(t-2)dt$ 6

   c) A tightly stretched string with fixed end points $x = 0$ and $x = l$, in the shape defined by $y = kx(l - x)$ where k is a constant, is released from this position of rest. Find $y(x,t)$, the vertical displacement if $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$. 8

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