Sem-III/Mech & Prod/CBGS/A-Maths-III/May 17

Q. P. Code: 11700

(3hours) [Total marks: 80]

N.B. (1) Question No. 1 is compulsory.
(2) Answer **any Three** from remaining
(3) Figures to the right indicate full marks

1. (a) State Cauchy Reimann equation in polar form. Find p if $f(z) = r^2 \cos 2\theta + ir^2 \sin p\theta$ is analytic. 5

(b) Find Laplace transform of $\sin 2t . \cos 3t$. 5

(c) Prove $\{\sin nx\}$, $n = 1, 2, 3, \ldots\ldots\ldots$ is orthogonal w.r.t. $(0, 2\pi)$. 5

(d) Evaluate $\int_{1+i}^{2+4i} (x^2 + ixy)\, dz$ along the curve $x = t, y = t^2$. 5

2. (a) Using Laplace transform, solve the differential equation,

$$\frac{dx}{dt} + 3x = 2 + e^{-t}, \text{ with } x(0) = 1.$$ 6

(b) Evaluate $\oint_C \frac{z+1}{z^3 - 2z^2} dz$ where $C : |z| = 1$. 6

(c) Obtain the Taylor's and Laurent series which represent the function $\frac{z^2 - 1}{(z+3)(z+4)}$ in the regions, (i) $|z| < 3$ (ii) $3 < |z| < 4$ (iii) $|z| > 4$. 8

3. (a) Solve $\frac{\partial^2 u}{\partial x^2} - 32 \frac{\partial u}{\partial t} = 0$ by Bender –Schmidt method, given $u(0,t) = u(x,0) = 0$, $u(1,t) = t$, taking $h = 0.25$. 6

(b) Evaluate- $\int_0^\infty t e^{-3t} \sin t\, dt$ 6

(c) Obtain Half Range Sine Series of $f(x) = x(\pi - x)$ in $(0, \pi)$.
Hence, evaluate $-\sum_{m=0}^{\infty} \frac{(-1)^m}{(2m+1)^3}$. 8

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4. (a) Find the orthogonal trajectory of the family of curves $x^3y - xy^3 = 0$. 6

   (b) Find Fourier series of $f(x) = |x|$ in $(-3, 3)$. 6

   (c) Find the inverse Laplace transform of the following:-

   (i) $\cot^{-1} s$ (ii) $\dfrac{8e^{-3s}}{s^2+4}$ 8

5. (a) Solve by Crank –Nicholson simplified formula $\dfrac{\partial^2 u}{\partial x^2} - \dfrac{\partial u}{\partial t} = 0$,

   $u(0,t) = u(1,t) = 0,\ u(x,0) = 100x(1-x)$

   taking $h = 0.25$ for one time step. 6

   (b) Using convolution theorem find the inverse Laplace transform of

   $$\frac{s}{(s^2+1)(s^2+4)}$$ 6

   (c) Find bilinear transformation which maps the points $z = 1, i, -1$ onto the points $w = i, 0, -1$. Hence, find the image of $|z| \le 1$ onto the w-plane. 8

6. (a) Using Residue theorem, evaluate, $\displaystyle\int_0^\infty \frac{dx}{x^2+1}$. 6

   (b) Obtain Complex form of Fourier series for $f(x) = e^{ax}$ over $-\pi < x < \pi$. 6

   (c) Determine the solution of one-dimensional heat equation $\dfrac{\partial u}{\partial t} = c^2 \dfrac{\partial^2 u}{\partial x^2}$ under boundary condition $u(0,t) = u(l,t) = 0,\ u(x,0) = x$, l being the length of rod. 8
