

Q.P. Code : 4755

(3 Hours)

[Total Marks : 80

- N.B. : (1) Question No. 1 is compulsory.
(2) Attempt any three questions out of remaining five questions.

- 1 (a) Find the Laplace transform of $te^{-t} \cosh 2t$ 5
- (b) Find the fixed points of $w = \frac{3z-4}{z-1}$. Also express it in the normal form. 5
- $\frac{1}{w-\alpha} = \frac{1}{z-\alpha} + \lambda$ where λ is a constant and α is the fixed point. Is this transformation parabolic?
- (c) Evaluate $\int_0^{1+i} (x^2-iy)dz$ along the path i) $y=x$, ii) $y=x^2$ 5
- (d) Prove that $f_1(x)=1$, $f_2(x)=x$, $f_3(x) = \frac{3x^2-1}{2}$ are orthogonal over $(-1,1)$ 5
2. (a) Find inverse Laplace transform of $\frac{2s}{s^4+4}$ 6
- (b) Find the image of the triangular region whose vertices are i , $1+i$, $1-i$ under the transformation $w = z+4-2i$. Draw the sketch. 6
- (c) Obtain fourier expansion of $f(x) = |\cos x|$ in $(-\pi, \pi)$. 8
3. (a) Obtain complex form of fourier series for $f(x)=\cosh 2x + \sinh 2x$ in $(-2,2)$. 6
- (b) Using Crank-Nicholson simplified formula solve $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$ given 6
- $u(0,t) = 0, u(4,t) = 0, u(x,0) = \frac{x}{3} (16-x^2)$ find u_{ij} for $i=0,1,2,3,4$ and $j=0,1,2$
- (c) Solve the equation $y + \int_0^t y dt = 1 - e^{-t}$ 8

4. (a) Evaluate $\int_0^{2\pi} \frac{d\theta}{5+3\sin\theta}$ 6
- (b) Find half - range cosine series for $f(x)=e^x$, $0 < x < 1$ 6
- (c) Obtain two distinct Laurent's series for $f(z) = \frac{2z-3}{z^2-4z-3}$ in powers of $(z-4)$ indicating the regions of convergence. 8
5. (a) Solve $\frac{\partial^2 u}{\partial x^2} - 2 \frac{\partial u}{\partial t} = 0$ by Bender - Schmidt method, given $u(0, t) = 0$, $u(4, t) = 0$, $u(x, 0) = x(4-x)$. Assume $h=1$ and find the values of u upto $t = 5$ 6
- (b) Find the Laplace transform of $e^{-4t} \int_0^t u \sin 3u \, du$ 6
- (c) Evaluate $\int_C \frac{z+3}{z^2+2z+5} dz$ where C is the circle i) $|z| = 1$, ii) $|z+1-i|=2$ 8
6. (a) Find inverse Laplace transform of $\frac{s}{(s^2-a^2)^2}$ by using convolution theorem. 6
- (b) Find an analytic function $\hat{f}(z) = u+iv$ where $u+v=e^x (\cos y + \sin y)$ 6
- (c) Solve the equation $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$ for the conduction of heat along a rod of length l subject to following conditions 8
- (i) u is not infinity for $t \rightarrow \infty$
- (ii) $\frac{\partial u}{\partial x} = 0$ for $x=0$ and $x=l$ for any time t
- (iii) $u=lx-x^2$ for $t=0$ between $x = 0$ and $x=l$